ASSESSMENT OF THE ERROR AND ADAPTIVITY FOR WAVE PROBLEMS

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ABSTRACT

This work presents an a posteriori estimator for the error in the wave number in the context of finite element approximations of the Helmholtz equation, both for standard and stabilized formulations. We also introduce a new goal-oriented adaptive strategy using post-processing techniques. The simple strategy assessing the error in the wave number is based on the determination of the numerical wave number that better accommodates the numerical solution. Compared to other goal-oriented error estimation strategies, the approach proposed in this work is innovative because it adopts a new paradigm. A distinctive feature of this method is that the error estimation procedure is devoted to obtain the numerical wave number, corresponding to the approximate solution, instead of the exact one, which is known as part of the data of the problem. Thus, the error in the wave number is consistently defined as the outcome of a global minimization problem. This problem is computationally unaffordable and, for practical error estimation purposes, is approximated. An enhanced approximation is obtained from the finite element solution using a simple local least-squares technique. Once the enhanced solution is obtained, the associated numerical wave number is readily recovered using a simple closed expression. An alternative improved recovery technique is developed to take advantage of the nature of the solutions of wave problems. The standard polynomial least-squares technique is replaced by a new exponential fitting, yielding much sharper results in most cases. The proposed new goal-oriented adaptive strategy is based on post-processed solutions and is valid both for linear and non-linear quantities of interest. In the non-linear case the linear contribution to the quantity of interest is assumed to be the leading term. Two different representations to recover the error in the quantity of interest are studied, both providing similar results in the adaptive procedures. It has been shown that the accuracy of these representations, which involve the post-processing of either the primal or adjoint finite element approximations, is related to the dispersion error of its corresponding problems. Moreover, the adaptive procedure leads to a faster reduction of the error when compared with a uniform refinement. The proposed error estimate properly identifies the areas most contributing to the error in the quantity of interest and consequently the adaptive procedure yields adapted meshes that provide accurate results.

Key words: *Wave problems, Helmholtz equation, Error estimation of wave number, Dispersion/pollution error, Goal-oriented adaptivity, Local indicators, Finite element method, Stabilized methods.*