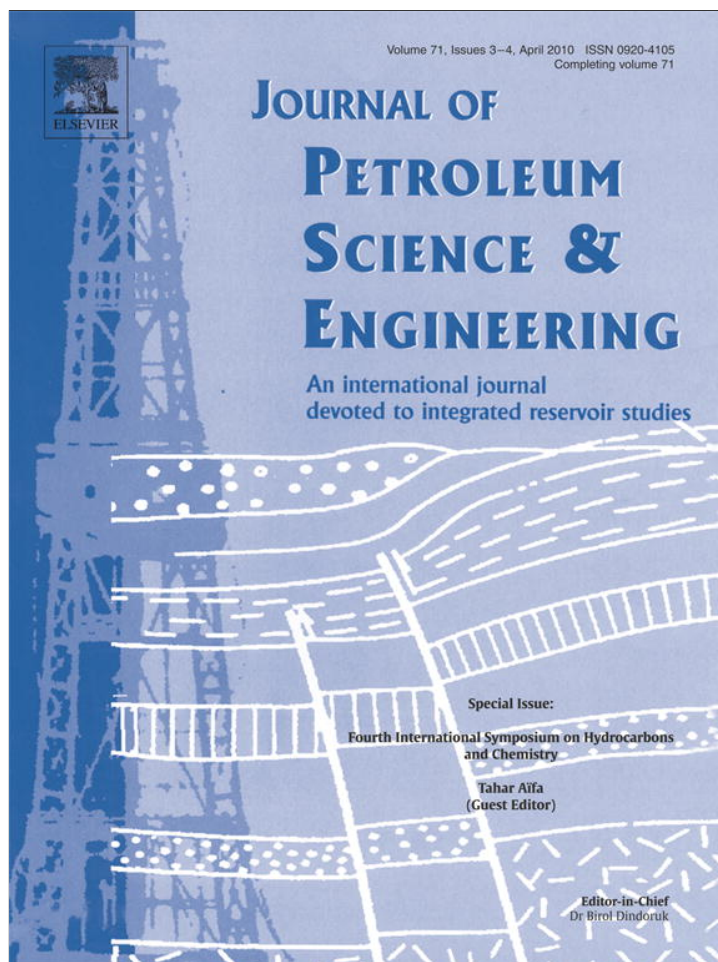


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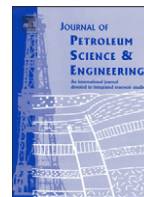
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## Research Paper

## A concurrent efficient global optimization algorithm applied to polymer injection strategies

Bernardo Horowitz\*, Leonardo José do Nascimento Guimarães, Vinicius Dantas, Silvana Maria Bastos Afonso

Department of Civil Engineering, Federal University of Pernambuco, Recife, PE, Brazil

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## ABSTRACT

One of the major difficulties in applying optimization to reservoir engineering problems is that each function evaluation requires a complete simulation which is computationally expensive. Moreover, some problems are known to be multimodal with several local minima. A common approach to tackle these problems is to construct cheap global approximation models of the responses often called metamodels or surrogates. These are based on simulation results obtained for a limited number of designs using data fitting. The optimization algorithm is coupled to the cheap metamodel. In this study a two-stage approach is employed based on the efficient global optimization algorithm, EGO, due to Jones. First an initial sample of designs is obtained using Latin hypercube. Parallel simulation runs for the initial sample are used to construct a Kriging metamodel. In the second stage the metamodel is used to guide the search for promising designs which are added to the sample in order to update the model until a suitable termination criterion is fulfilled. The selection of designs which are adaptively added to the sample is based on the maximization of the expected improvement merit function which balances the need for improving the value of the objective function with that of improving the quality of the metamodel prediction. In this study the original EGO algorithm is modified to exploit parallelism. The modified algorithm is applied to a polymer injection optimization problem. This eight-variable problem maximizes economical return by controlling the starting time and slug duration in each injector well. In the presented example a parametric study was conducted varying oil price. It is concluded that polymer flooding is feasible for oil prices above US\$20.00/STB and gains increase with oil price.

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## 1. Introduction

Polymer injection in petroleum reservoirs has been used worldwide for decades where polymers act basically by increasing the injected water viscosity. The management of this Enhanced Oil Recovery (EOR) method requires determination of optimal injection planning based on reservoir simulation. The injection planning consists in determining concentration of polymer in water and, for each injector, time of injection beginning and duration of slugs. This can be formulated as an optimization problem whose objective function is the economical return of the proposed EOR method as compared to traditional waterflooding scheme. As it is a multimodal optimization problem, a global search strategy should be preferably used to solve it.

Every trial of a polymer injection design requires a complete reservoir simulation for its assessment. As simulations are computationally expensive, their direct use in optimization algorithms is infeasible. A common approach to tackle this problem is to construct cheap global

approximation models of the responses often called metamodels or surrogates. These are based on simulation results obtained for a limited number of designs using global data fit techniques such as Kriging applied to deterministic computer code outputs commonly called DACE (Design and Analysis of Computer Experiments) (Sacks et al., 1989). Computation of all necessary function values required by the optimization algorithm is performed using the cheap metamodel. The ability of the metamodel to tame general nonsmoothness of the simulation responses is recognized (Eldred and Dunlavy, 2006). Should nonsmoothness become more significant specific techniques may have to be employed (Forester et al., 2006).

In this study a two-stage approach is employed based on the efficient global optimization algorithm, EGO (Jones, 2001). The EGO algorithm has been recently applied to reservoir engineering for reservoir characterization as well as EOR (Queipo et al., 2000, 2002; Zerpa et al., 2005). First an initial sample of designs is obtained using some design of experiments (DOE) technique, such as Latin hypercube (Giunta et al., 2003). Simulation runs for the design points are used to construct the metamodel. In the second stage the metamodel is used to guide the search for promising designs which are then added to the sample in order to update the metamodel until a suitable termination criterion is fulfilled. The selection of designs to be added to the sample is called infill sampling criterion (ISC) (Sasena, 2002). ISC should

\* Corresponding author. Departamento de Engenharia Civil, UFPE, Av. Acaemico Hélio Ramos, s/n, Recife, PE 50.740-530, Brazil. Tel.: +55 81 91890091; fax: +55 81 32222540.

E-mail address: [horowitz@ufpe.br](mailto:horowitz@ufpe.br) (B. Horowitz).

balance the need of improving the value of the objective function with that of improving the quality of the prediction so that one does not get trapped in a local minimum. Considering the computational cost of the simulation and the availability of parallel computing it is highly desirable for the ISC to include in each iteration multiple promising designs whose simulation may be performed concurrently. In this study the ISC proposed by Jones et al. (1998) that uses the expected improvement merit function is modified to exploit parallelism.

Initially the polymer injection problem is described and then the optimization formulation is given. The optimization strategy is described starting with the construction of the DACE surrogate model followed by the original EGO algorithm and the proposed parallel ISC. Finally, an example application is presented to demonstrate the potentials of the proposed methodology.

## 2. The polymer injection problem

Polymer flooding uses high molecular-weight polymer to improve waterflooding performance by increasing the viscosity of the injected water thereby resulting in more favorable mobility ratios. It has been demonstrated to accelerate oil production while delaying water breakthrough, resulting in a higher recovery without affecting the residual oil saturation. The major aspects to take into account in numerical modeling are the mobility control and polymer retention (Kaminsky et al., 2007).

The main beneficial effect is the increase in water viscosity that is a function of polymer concentration in water. Another important aspect to mobility control is the reduction in absolute permeability due to the mechanism of polymer retention in reservoir rock. In the model, this reduction is a function of concentration of polymer retained by the rock. The polymer retention effects are due to two different mechanisms: adsorption by the rock surface and blocking of smaller caliber pores by the polymer molecules, resulting in an inaccessible porous volume to the fluids. From the experimental point of view it is difficult to quantify these two mechanisms separately. Therefore the retention effects are modeled by a single non-linear adsorption isotherm. The mass conservation equations of the problem are those of blackoil model modified to include the polymer mass transport. Polymer reduced porosity is a fraction of the original rock porosity modeled by the inaccessible pore volume constant,  $IPV$ :  $\bar{\phi} = (1 - IPV)\phi$ . The modified absolute permeability tensor is a fraction of the original tensor which depends on the residual resistance factor,  $RRF$ , and the ratio between actual adsorbed polymer concentration and maximum adsorptive capacity of the rock,  $A_{dMAX}$ . A linear model is adopted for water viscosity which is a function of polymer concentration in the water phase.

## 3. Problem definition

In this work the chemical choice and concentration, as well as injection water rates, are considered fixed. The design variables for each injector are the starting time and slug duration:

$$\left. \begin{aligned} x_{2i-1} &= \text{starting time for well } i \\ x_{2i} &= \text{slug duration for well } i \end{aligned} \right\} i = 1 \dots niw \quad (1)$$

where  $niw$  = number of injector wells. In order to define the objective function, let:

$$\text{Base Case} \begin{cases} CWI_0 = \text{cumulative water injection} \\ COP_0 = \text{cumulative oil production} \end{cases} \quad (2)$$

The Base Case is the reference where the water injection rates as well as well constraints are kept the same for the optimization problem simulation except no polymer is injected. A simple objective function used in this study is given by:

$$RI(x) = (COP(x) - COP_0) \cdot op - (CWI(x) - CWI_0) \cdot wic - CPI(x) \cdot pc \quad (3)$$

where:  $RI(x)$  = relative improvement of injection schedule;  $COP(x)$ ,  $CWI(x)$  = cumulative oil production and cumulative water injection from simulation;  $CPI(x)$  = cumulative polymer injection;  $op$  = oil price;  $wic$  = water injection cost and  $pc$  = polymer cost. An alternative objective function taking into account discounted cash flow values is given by:

$$NPVRI(x) = \sum_{\tau=0}^T \left[ \frac{1}{(1+d)^\tau} F_\tau(x) \right]$$

with :  $F_\tau(x) = (OP_\tau(x) - OP_{\tau_0}) \cdot op - (WI_\tau(x) - WI_{\tau_0}) \cdot wic - PI_\tau(x) \cdot pc$  (4)

where:  $F_\tau(x)$  = cash flow;  $OP_\tau$ ,  $WI_\tau$ ,  $PI_\tau$  = oil production, water injection, and polymer injection in time interval  $\tau$ ;  $d$  = discount rate.

The optimization problem can be formulated as:

$$\begin{aligned} &\text{Maximize } OF(x) \\ &\text{subject to : } x_{2i-1} + x_{2i} \leq cp, \quad i = 1 \dots niw \\ &\quad \quad \quad x \geq 0 \end{aligned} \quad (5)$$

where:  $OF(x)$  = objective function in use,  $RI(x)$  or  $NPVRI(x)$ ; and  $cp$  = concession period. Well constraints such as maximum/minimum bottom hole pressure BHP and maximum/minimum fluid rates are handled by the reservoir simulator and become hidden constraints for the optimizer. This may result in objective functions that are not continuously differentiable with respect to design variables. This should not be necessarily a problem for the proposed optimization solver since it is not gradient-based, and uses smooth surrogate data fitting models.

## 4. Optimization strategy

As mentioned above the optimization strategy adopted in this study is based on the efficient global optimization algorithm, EGO (Jones et al., 1998), modified by a proposed parallel infill sampling criterion. The strategy is described in the following sections starting out by the construction of the DACE metamodells followed by the original EGO infill sampling criterion. The EGO algorithm is then briefly described and finally the proposed parallel infill sampling criterion motivation and implementation are presented.

### 4.1. DACE metamodells

Metamodels construction typically involves one of the following strategies: data fitting schemes (Giunta and Watson, 1998; Simpson et al., 2001; Keane and Nair, 2005), Taylor series expansions (Giunta and Watson, 1998; Giunta and Eldred, 2000), and reduced basis (Afonso and Patera, 2003). Data fit type surrogates typically involve interpolation or regression (polynomial) of a set of data generated from the high-fidelity model. The regression models present two major drawbacks for a meta-model construction: (1) the difficulty to specify the regression terms as the functional form of the high-fidelity function is unknown a priori; (2) the assumption of independent errors (do not consider the correlation between the points).

Interpolation models commonly used as surrogates are based on techniques known as Kriging, a well-known stochastic based process model in the field of statistics and geostatistics, which started by the late eighties to be used as approximation technique of outputs obtained from deterministic computer simulations. Kriging models (Jones et al., 1998; Simpson et al., 2001; Gano and Renaud, 2004) differ from regression models in the sense that they in general give a global approximation to the response and also can capture oscillatory response trends. Moreover, the sample values are assumed to exhibit spatial correlation with response values modeled via a Gaussian process around each sample location. After the unknowns are estimated some validation checks need to be conducted to judge the quality of the generated substitute model to

be used. This work will focus on Kriging type of approximation. The main aspects of this methodology are described next.

4.1.1. Design of experiments (DOE)

The first step on the construction of a data fitting based metamodel is to generate the sampling points. These are unique locations in the design space determined by a design of experiments (DOE) approach (Giunta and Watson, 1998; Keane and Nair, 2005). In such locations the response values of the high-fidelity model are obtained to construct the approximated model (by instance, Kriging interpolation is very much influenced by the sampling location). The samplings selection is a very important stage to build a reliable metamodel. Specifically for high computational cost function evaluations one must seek for an effective sampling plan, which means the minimum number of points that ensure a metamodel with good accuracy.

Commonly considered approaches are Monte Carlo, Quasi Monte Carlo, Latin hypercube sampling (LHS), orthogonal array, centroidal voronoi tessellation (CVT) (Giunta and Watson, 1998; Keane and Nair, 2005). In this work LHS sampling will be used throughout. To obtain a LHS sampling, the range of each variable is divided into  $p$  “bins” (sub-intervals) of equal probability. For  $n$  design variables this partitioning yields a total of  $p^n$  subintervals in the parameter space. Next,  $p$  samples are randomly selected in the design domain space under certain restrictions such as: each sample is randomly placed inside a domain partition and for each unidimensional projection ( $x_i$ ) of the samples and partitions, there will be one and only one sample in each partition (Giunta et al., 2003). The above explanation is easily perceived in Fig. 1 in which four samples are to be placed in a 2D ( $x_1, x_2$ ) domain. For this particular case  $p = 4$ , consequently four partitions are placed in both  $x_1$  and  $x_2$  axes. This gives a total of 16 bins of which four will be chosen satisfying both restrictions. Bullets in Fig. 1 represent the four sample sites chosen randomly in each bin.

The randomness inherent in the procedure means that there is more than one possibility of arrangement of sampling that meet the LHS criteria. As the LHS sampling is stochastic in nature, it is advised to run such scheme several times and select the best sampling for usage. This can be automatically calculated following the suggestion given by Keane and Nair (2005) in which for each LHS a quantity  $\Delta$  is obtained as:

$$\Delta = \left( \sum_{i=1}^{m-1} \sum_{j=i}^m \frac{1}{\sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}} \right) \quad (6)$$

where  $m$  is the total number of samplings. According to Keane and Nair (2005), the LHS distribution that gives the minimum value for  $\Delta$  is the selected sample.

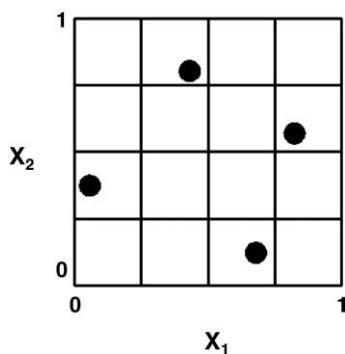


Fig. 1. Latin hypercube four samples example in a 2D ( $x_1, x_2$ ) domain.

4.1.2. Kriging formulation

In this technique the following model is considered to the true unknown function

$$f(x) = \sum_{j=1}^k \beta_j N_j(x) + \varepsilon(x). \quad (7)$$

In the above equation the first part is a linear regression of the data with  $k$  regressors, in which  $\beta_j$  ( $j = 1 \dots k$ ) are the unknowns and  $\varepsilon(x)$ , the error, is responsible to create a ‘localized’ deviation from the global model. As previously mentioned, polynomials are generally used to construct  $N_j(x)$ . A traditional approach is called ordinary Kriging in which zero order (constant) function is employed.

In the Kriging process a correlation between errors related to the distance between the corresponding points is assumed. Different forms of correlation functions may be employed (Giunta and Eldred, 2000). In this work the following Gaussian correlation form is assumed

$$\text{Corr}(x^{(i)}, x^{(j)}) = \exp[-d(x^{(i)}, x^{(j)})] \quad (8)$$

in which  $d(x^{(i)}, x^{(j)})$  is a special normalized distance given by:

$$d(x^{(i)}, x^{(j)}) = \sum_{k=1}^n \theta_k \left| x_k^{(j)} - x_k^{(i)} \right|^{p_k} \quad (\theta_k \geq 0; p_k \in [1, 2]). \quad (9)$$

Where  $n$  is the total number of variables,  $\theta_k$  are the unknown correlation parameters used to fit the model which measures the importance or activity of  $x_k$  and  $p_k$  relates the smoothness of function in terms of variable  $x_k$  (Jones et al., 1998). In this work  $p_k = 2$  is considered. As already pointed out, this correlation form is so powerful that a simple constant term for the regression part of Eq. (7) (ordinary Kriging) can be used in substitution of the model previously presented such as

$$f(x^{(i)}) = \mu + \varepsilon(x^{(i)}) \quad (i = 1, \dots, m) \quad (10)$$

where  $\mu$  is the mean of the stochastic process and  $\varepsilon(x^{(i)})$  is Normal( $0, \sigma^2$ ).

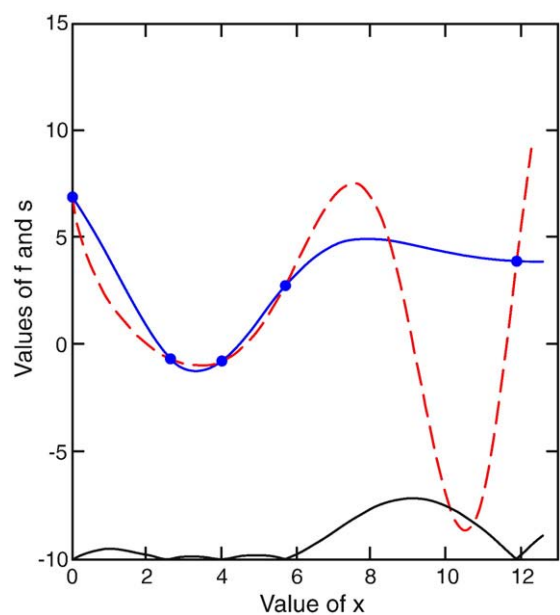


Fig. 2. Initial DACE model.



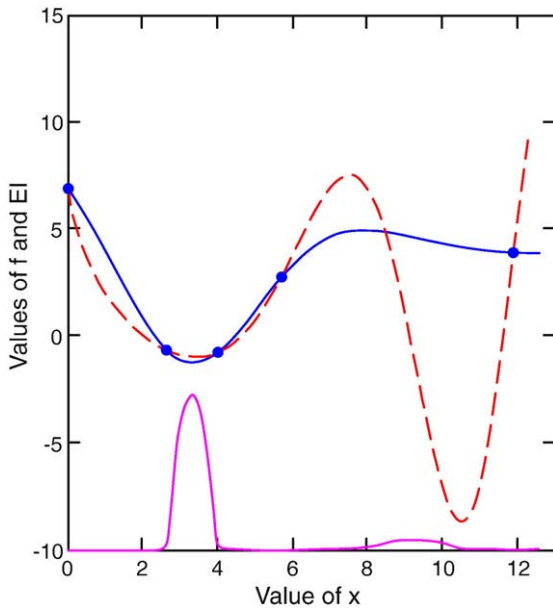


Fig. 3. Expected improvement function.

In the Kriging model a total of  $k + 2$  unknowns ( $\mu, \sigma^2, \theta_1, \dots, \theta_k$ ) have to be obtained to compose the approximation. Such parameters are estimated by maximizing the likelihood function,  $lf$ , given by

$$lf = \frac{1}{(2\pi)^{n/2} (\sigma^2)^{n/2} |\mathbf{R}|^{1/2}} \exp \left[ -\frac{(f - \mathbf{1}\mu)' \mathbf{R}^{-1} (f - \mathbf{1}\mu)}{2\sigma^2} \right] \quad (11)$$

in which  $\mathbf{f} = (f^{(1)}, \dots, f^{(m)})^T$  is the true function at the samplings;  $\mathbf{1} = (1, \dots, 1)^T$  and  $\mathbf{R}$  is a  $m \times m$  correlation matrix with unity values along the diagonal whose  $(i, j)$  entry is  $\text{Corr}[\varepsilon(\mathbf{x}^{(i)}), \varepsilon(\mathbf{x}^{(j)})]$  between any two of the  $m$  sampled data points  $\mathbf{x}^{(i)}$  and  $\mathbf{x}^{(j)}$ .

The maximization of the  $lf$  function (Eq. (11)) leads to (Jones et al., 1998):

$$\hat{\mu} = \frac{\mathbf{1}' \mathbf{R}^{-1} \mathbf{f}}{\mathbf{1}' \mathbf{R}^{-1} \mathbf{1}} \text{ and } \hat{\sigma}^2 = \frac{(f - \mathbf{1}\hat{\mu})' \mathbf{R}^{-1} (f - \mathbf{1}\hat{\mu})}{m} \quad (12)$$

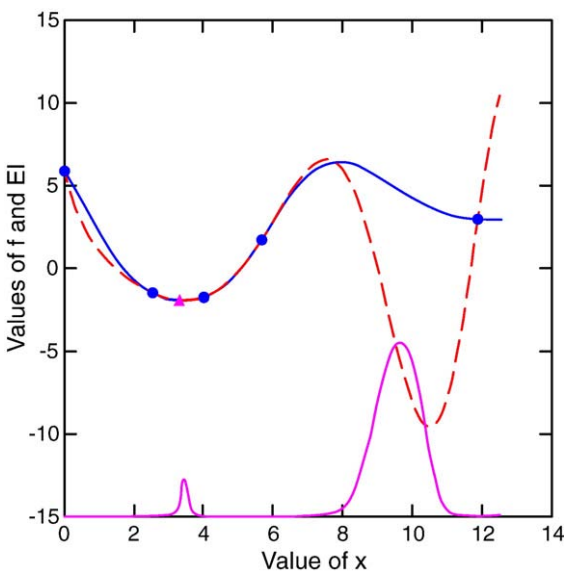


Fig. 4. Functions after addition of first local maximum.

Under the application of the above estimates into Eq. (11) and the maximization of  $lf$  function the remaining unknowns are obtained (correlation parameters  $\theta$ ). After that the best linear unbiased predictor (BLUP) at any point of the design domain can be obtained as

$$\hat{f}(x) = \hat{\mu} + r^T \mathbf{R}^{-1} (f - \mathbf{1}\hat{\mu}) \quad (13)$$

in which  $r(x)$  is a correlation vector, which correlates an untried  $x$  and  $m$  sampled data points. Kriging formulation also allows obtaining the mean squared error of the predictor at any untried point as follows (Jones et al., 1998):

$$s^2(x^*) = \sigma^2 \left[ 1 - r^T \mathbf{R}^{-1} r + \frac{(\mathbf{1} - \mathbf{1}' \mathbf{R}^{-1} r)^2}{\mathbf{1}' \mathbf{R}^{-1} \mathbf{1}} \right] \quad (14)$$

It will be convenient to work hereafter with the square root of the mean squared error,  $s = \sqrt{s^2(x)}$ , denoted by RMSE.

In order to assess the adequacy of the Kriging model we use the PRESS (prediction residual error sum of squares) error measure also referred to as “leave one point out” cross validation method (Keane and Nair, 2005). It assesses the accuracy of the model when individual data points are omitted from the data used to create the approximation:

$$PRESS = \sum_{i=1}^m (f_i - \hat{f}_i)^2 \quad (15)$$

where  $\hat{f}_i$  is the  $i$ th function approximation model obtained by omitting the  $i$ th data point from the data set.

#### 4.2. Infill sampling criterion

The second stage of the metamodel based approach used in this study searches at each iteration for promising design points to enter the training sample set. The simplest approach would be to choose the minimizer of the predictor itself. This strategy would put too much emphasis on the local behavior of the objective function which would force convergence to the local minimum closest to the predictor minimizer. This can be easily demonstrated in the example shown in Fig. 2 where the exact, true, function is depicted in a dashed line while

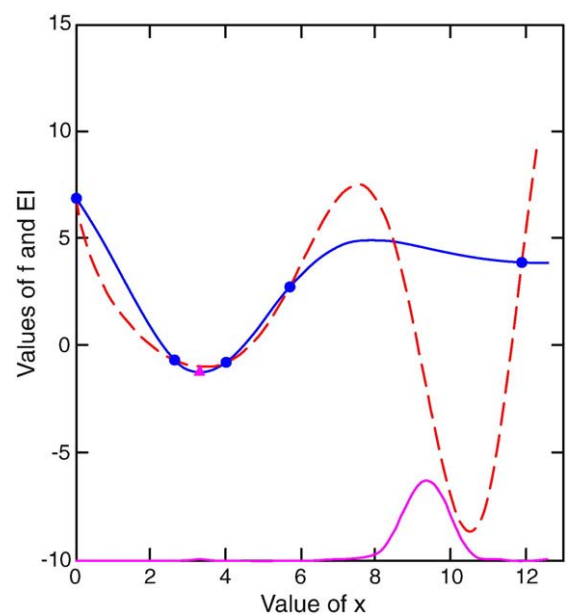


Fig. 5. Functions after addition of first promising point to temporary sample.

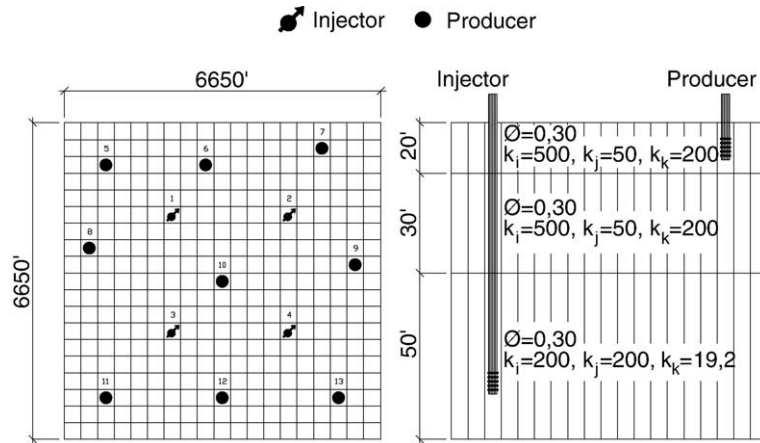


Fig. 6. Example oil field (permeabilities in mD).

the Kriging approximation based on five samples is plotted with a continuous line. There is a larger concentration of samples on the left which can easily happen in the case of larger number of variables, especially when some new designs are added to the sample. Also plotted on the bottom of the figure is the value of the RMSE. The standard error is zero at the sampled points and larger in the poorly sampled right half of the plot. If one chooses the predictor minimizer to enter the sample it is clear that the process will eventually converge to the local minimum on the left, missing the global minimum of the objective function.

The example shows that one should not concentrate only on improving the value of the objective function but also on improving the accuracy of the predictor. In the latter case designs with large prediction uncertainty should also be included in the training sample. In order to have a balanced approach one should also search for points where the probability of improvement is higher (Jones et al., 1998). Let  $f_{\min}$  be the current best value of the objective function:

$$f_{\min} = \text{Min}\{f(x_1), f(x_2), \dots, f(x_m)\}. \quad (16)$$

The improvement,  $I(x)$ , over  $f_{\min}$  at  $x$  is defined by:

$$I(x) = \text{Max}\{f_{\min} - \hat{f}(x), 0\}. \quad (17)$$

If one assumes that  $\hat{f}(x)$  is a realization of a Gaussian process there is some probability of its improvement upon  $f_{\min}$ . If one weights the improvement by the associated probability one gets the “expected improvement” function:

$$EI(x) = E[I(x)] \quad (18)$$

which is the expected value of the improvement at  $x$ . It can be shown that (Jones et al., 1998):

$$EI(x) = [f_{\min} - \hat{f}(x)] \Phi \left[ \frac{f_{\min} - \hat{f}(x)}{s} \right] + s \varphi \left[ \frac{f_{\min} - \hat{f}(x)}{s} \right] \quad (19)$$

where:  $\Phi$  = standard normal cumulative distribution function;  $\varphi$  = standard normal density function and  $s$  is defined in Section 4.1.2.

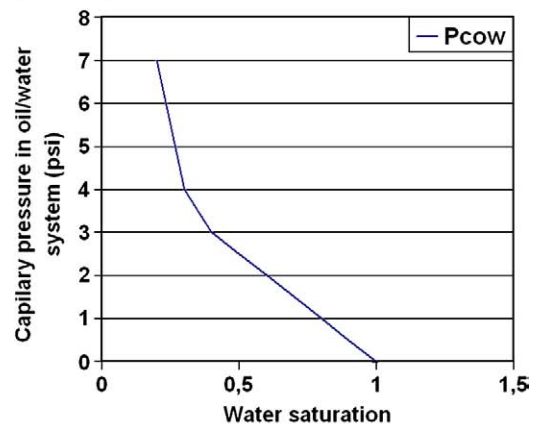
The first term is the predicted difference between current minimum and  $\hat{f}(x)$  weighted by the probability of improvement (Schonlau, 1997). Hence it is large where  $\hat{f}(x)$  is likely to be smaller than  $f_{\min}$ . The second term is large when  $\hat{f}(x)$  is close to  $f_{\min}$  and  $s$  is large denoting much uncertainty with the prediction. Therefore the expected improvement function strikes a balance between exploiting regions of the design space where good solutions have been found and

exploring regions that are undersampled thus having greater uncertainty (Bichon et al., 2007).

A plot of the expected improvement function is shown on the bottom of Fig. 3. It can be seen that it has two local maxima: one in the region of expected function decrease and another in the poorly sampled area.

The optimization of the expected improvement will add to the sample a point very close to the local minimum of the true objective

### (a) Capillary pressure



### (b) Relative permeability

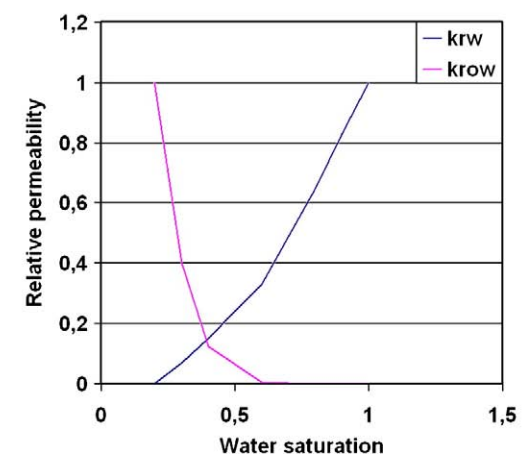


Fig. 7. Capillary pressure and relative permeability curves.

**Table 1**  
Reservoir characteristics.

Property	<i>i</i>	<i>j</i>	<i>k</i>
Size of cells (ft)	350	350	20, 30, 50
Permeabilities (1st layer) (mD)	500	500	200
Permeabilities (2nd layer) (mD)	50	50	200
Permeabilities (3rd layer) (mD)	200	200	19.2
Porosity	0.30		
Reference level for pressure (ft)	4150		
Rock compressibility (1/psi)	$3.0 \times 10^{-6}$		
Reservoir pressure (psi)	3000.0		
Bubble point pressure (psi)	2500.0		

function. But once this new point enters the training sample set the resulting expected improvement function attains a maximum close to the global minimum, as can be seen from Fig. 4. Now it can be readily perceived that the addition of this new design to the sample will ensure final convergence to the true global minimum.

As it can be observed from Figs. 3, and 4 the expected improvement function vanishes at sampled points and tend to be highly multimodal (Jones et al., 1998). The local maxima of the expected improvement function are at designs with the highest probability of function decrease or predictor uncertainty. Therefore those designs are promising points to enter the sample in order to better update the metamodel (Sobester et al., 2005).

4.3. The EGO algorithm

The algorithm proposed by Jones et al. (1998) is schematically described below:

- (1) Generate a small number of samples from the objective function:
  - a) An initial sample of  $m = 10n$ , where  $m$  = initial sample size and  $n$  = number of variables (Jones et al., 1998) has been suggested. Others propose  $m = (n + 1)(n + 2)/2$ , the number of points necessary to fit a full quadratic polynomial (Bichon et al., 2007).
  - b) Latin hypercube or other sampling technique is used to generate the initial sample (Giunta et al., 2003).
- (2) Construct an ordinary Kriging based metamodel from initial sample:
  - a) Other technique such as radial basis functions may also be used (Sobester et al., 2005).
  - b) The metamodel is crossvalidated by leaving one observation at a time out and then predicting it based on the remaining sample points. If crossvalidation fails a log or inverse transformation is tried.
- (3) Find the design that maximizes the expected improvement function.
- (4) If maximum improvement is less than  $TOL * f_{min}$ , stop. The suggested value for TOL is 1% but this value may have to be reviewed if log or inverse transformation was applied (Jones et al., 1998).
- (5) Add new design to sample and update metamodel. Go to step [3].

**Table 2**  
Oil and water properties.

Property	Oil	Water
Density (lbm/ft <sup>3</sup> )	46.244	62.238
Compressibility (1/psia)	$1.3687 \times 10^{-5}$	$3.04 \times 10^{-6}$
Formation volume factor (RB/STB)	1.50	1.04
Viscosity (cp)	1.04	0.31

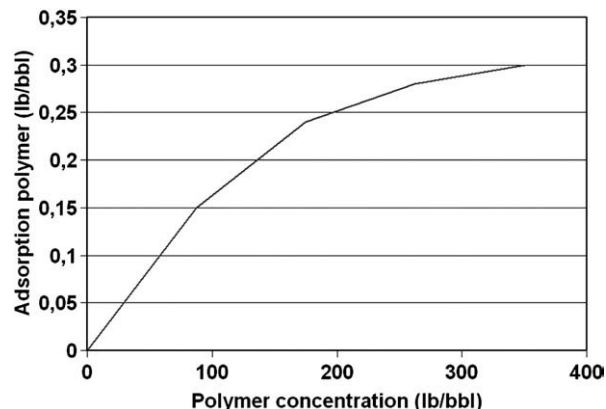


Fig. 8. Polymer adsorption isotherm for a rock with permeability of 10 mD.

As the computation cost of reservoir simulation is high and the availability of parallel computation has increased dramatically in the oil industry setting it is highly desirable to include multiple designs in the ISC. In fact, as has been suggested earlier, local maxima of the expected improvement function are generally promising points whose addition to the sample may increase efficiency of the algorithm.

4.4. Parallel infill sampling criterion

We propose below a parallel infill sampling criterion. It is motivated by the following observations:

- (1) Local maxima of the expected improvement function are designs where either there is a high probability of objective function decrease or high predictor uncertainty. These are promising designs to improve the objective function as well as metamodel predictive capability.
- (2) As one includes a local maximum of the expected improvement in the sample the value of the updated expected improvement drastically reduces in the neighborhood of the point and another local maximum becomes dominant as observed in Fig. 4.

Let  $n_p$  be the number of available processors and  $n_d \leq n_p$  be the number of promising designs to enter the sample. The proposed change to step [3] of EGO algorithm is given below.

Copy current sample set to temporary working sample set.  
 For  $i = 1 \dots n_d$   
 Maximize the expected improvement function, obtaining  $x_i^*$ .  
 Append the pair  $(x_i^*, \hat{f}(x_i^*))$  to temporary working sample.  
 Temporarily update Kriging metamodel.

Next  $i$ .

The following remarks further detail the proposed implementation:

- (1) DIRECT algorithm (Finkel, 2003) is used to optimize the expected improvement function. It is a derivative-free global optimizer that reaches a solution by selecting and subdividing at each iteration hyper-cubes that are most likely to contain the global optimum.

**Table 3**  
Permeability related polymer constants.

Permeability (mD)	$A_{dMAX}$ (lb/bbl)	IPV	RRF
10.0	0.30	0.05	1.20
1000.0	0.20	0	1.20

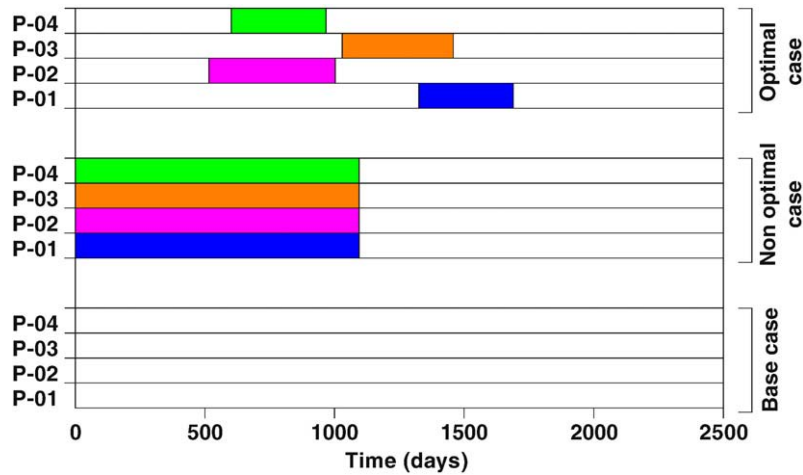


Fig. 9. Starting injection time and slug duration.

- (2) Observe that the true function is never invoked to compute functional values at new sample points. The cheap metamodel is always used instead.
- (3) As Kriging predictor is used to compute  $\hat{f}(x_i^*)$  there is no need to recompute maximum likelihood parameters for metamodel updating.
- (4) In Fig. 5 the maximizer of the expected improvement function evaluated using the Kriging predictor is added to temporary working sample and the metamodel is updated. Note that the added point lies on the continuous line representing the approximating metamodel. Note also that the resulting function filters out the added solution so that the global optimizer may find the next local maximum.
- (5) Correlation matrix  $\mathbf{R}$  becomes ill-conditioned when samples get clustered around a given design. This happens because rows and columns of  $\mathbf{R}$  become almost identical (Sasena, 2002). Therefore we use the following safeguard when adding a point to the

sample: if minimum distance of new point to the existing sample is less than  $tol$ , then perturb the new point in a random direction so that its distance from the nearest point found varies randomly from one to ten times  $tol$ . The value used for  $tol$  is:

$$tol = 10^3 \sqrt{\varepsilon_M} \quad (20)$$

where:  $\varepsilon_M$  = machine epsilon.

- (6) In step [5] of the EGO algorithm we add the  $n$  additional design points to the sample where true functions are concurrently evaluated to update the metamodel for next iteration.

### 5. Example application

Consider the field shown in Fig. 6, containing four injectors and nine producer wells, with property data based on IMEX template MXSPR005 (CMG, 2007) and geometry similar to Zerpa et al. (2005),

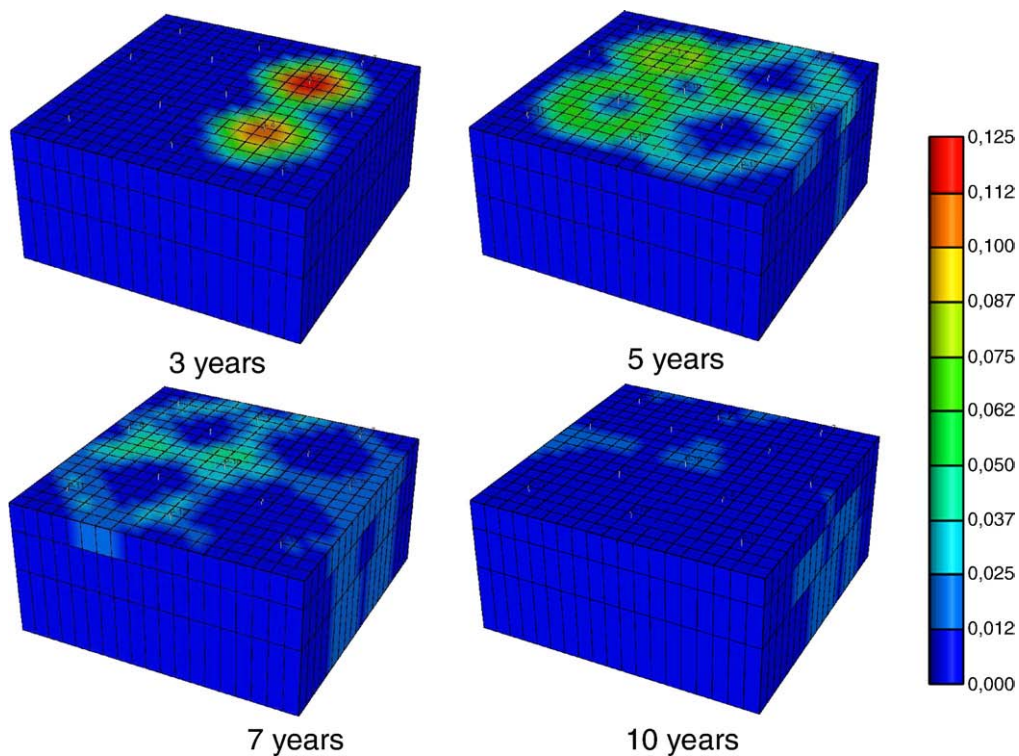


Fig. 10. Evolution of polymer concentration.



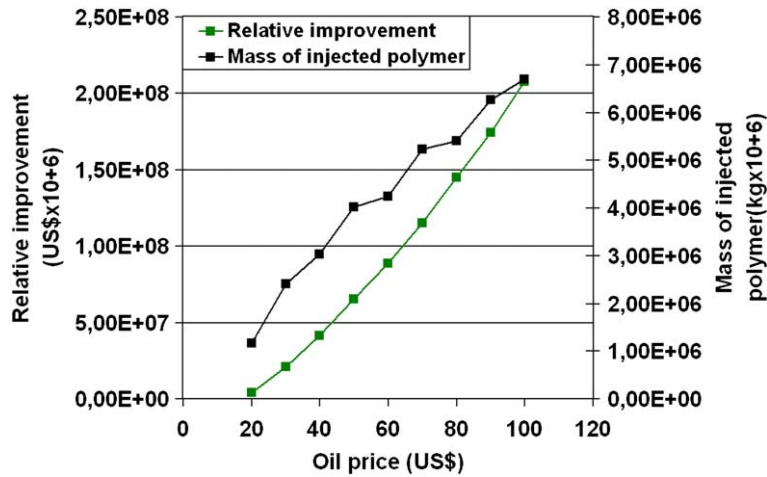


Fig. 11. Relative improvement and mass of injected polymer as functions of oil price.

modeled with 1083 cells. Polymer is injected at a constant concentration of 0.7 lb/STB while the maximum water injection rate is 10,000 STB/day at a maximum BHP of 9000 psi during the whole simulation period of ten years. Producers operate at a maximum fluids rate of 2500 STB/day and minimum BHP of 1500 psi.

In all studied cases the reservoir is initially under-saturated (pressure above the bubble point) with an initial oil saturation  $S_o = 0.8$  ( $S_w = 0.2$ ). The adopted capillary curve and relative permeabilities are shown in Fig. 7.

Tables 1 and 2 detail the rock and fluids parameters adopted in the simulations. The adsorption of polymer by the rock as a function of polymer concentration in water is shown in Fig. 8 as a non-linear adsorption isotherm for a rock with permeability of 10 mD. For different rock permeabilities linear interpolation of rock/polymer parameters from Table 3 may be used.

We initially consider three cases: base, non optimal and optimal. In the base case only water is injected. Polymer is injected for the first three years at each injector for the non optimal case.

The optimization problem formulated as described above has eight variables. The  $RI$  function is computed using  $op = US\$70/STB$ ,  $wic = US\$0.29/STB$  and  $pc = US\$15.15/STB$ . The starting time and slug duration for the non optimal and the obtained solution are shown in Fig. 9. The distribution of injected polymer in reservoir at different times is shown in Fig. 10 for the optimal solution. The different injection starting times for the injectors and the fast spreading of polymer in the reservoir can be recognized. Seven additional samples were used in the proposed parallel infill sampling criterion. The algorithm converged in 21 iterations, requiring 248 simulation runs, with an optimal  $RI$  value of  $US\$115.2 \times 10^6$ , corresponding to an increase of 2.76 MMSTB in oil

production and a decrease in water production of 2.95 MMSTB, relative to the non optimal case. Using the original EGO algorithm, without additional samples, the number of iterations increases by a factor of 2.33, doubling the required clock time to obtain the solution. The non optimal case results in  $RI = US\$26.3 \times 10^6$ , which is 23% of the optimal value.

In order to assess the sensitivity of the optimal polymer injection solution relative to oil price, A parametric study for nine oil prices from  $op = US\$20/STB$  to  $US\$100/STB$  was conducted. Relative improvement and total mass of injected polymer variations with oil price are shown in Fig. 11. The sharp increase of both values with increase in oil prices is readily appreciated. Some insight can be gained from Fig. 12, which shows the variations of oil and water cumulative productions as functions of oil price. It is clear that polymer injection method becomes increasingly feasible with higher oil prices. This is corroborated by the increase in size of the polymer slugs with oil price as depicted in Fig. 13 for the cases of  $op = US\$40/STB$ ,  $US\$70/STB$  and  $US\$100/STB$ . It is also worth noting that optimal injection strategies starts out injecting water only, followed by polymer injection after the second year of exploitation. Also, polymer injection concentrates at the first years of the concession period rather than at the end. When objective function  $RI(\mathbf{x})$  of Eq. (3) is exchanged for  $NPVRI(\mathbf{x})$  of Eq. (4), with a mean annual rate  $d = 9.3\%$ , optimum results change slightly with an average reduction of 0.3% in the final cumulative oil production, and 14.7% in the total mass of injected polymer.

## 6. Conclusions

In reservoir engineering problems, each functional evaluation requires a complete model simulation which is computationally expensive. Some of these problems are also multimodal with several

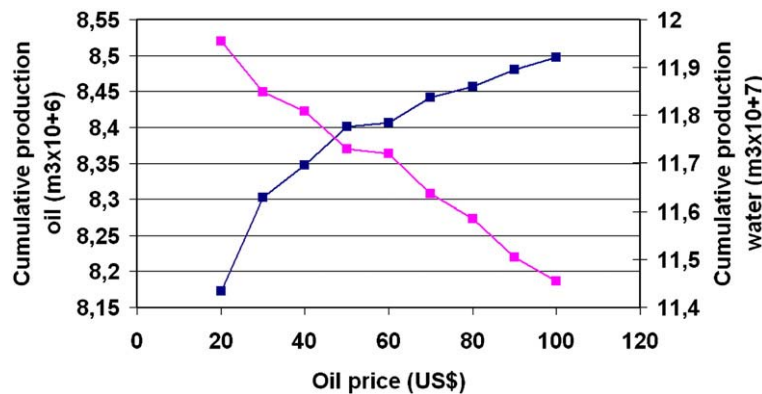


Fig. 12. Oil and water cumulative productions as functions of oil price.

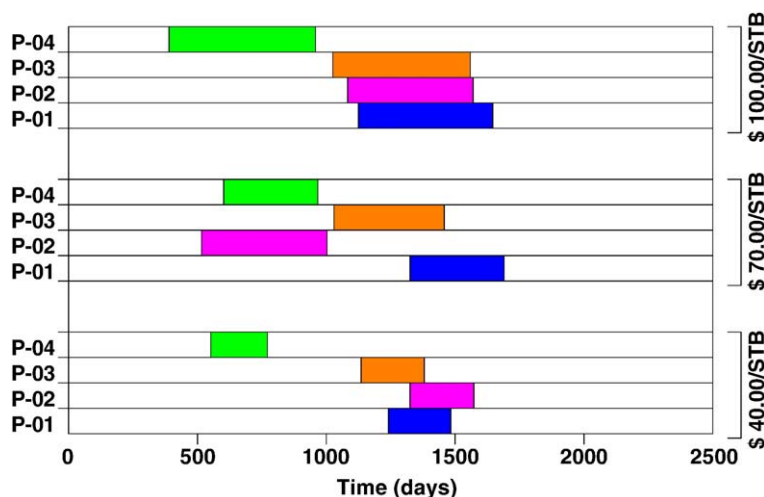


Fig. 13. Initial injection time and slug duration for different oil prices.

local minima. A common approach to tackle these problems is to construct cheap global approximation models of the responses, often called metamodels or surrogates.

In this study a two-stage approach is employed based on the efficient global optimization algorithm, EGO, due to Jones. First an initial sample of designs is obtained using a design of experiments technique. Parallel simulation runs for the initial sample are used to construct a Kriging metamodel. In the second stage the metamodel is used to guide the search for promising designs which are added to the sample in order to update the model until a suitable termination criterion is fulfilled. The selection of designs which are adaptively added to the sample should balance the need for improving the value of the objective function with that of improving the quality of the prediction so that one does not get trapped in a local minimum. In the EGO algorithm this balance is achieved through the use of the expected improvement merit function.

The original EGO algorithm is modified to exploit parallelism by selecting some local minima of the merit function to enter the sample at each iteration. The modified algorithm is applied successfully to a polymer injection optimization problem. The advantages of adopting the proposed parallel infill sampling criterion are noticeable: the total number of iterations is decreased, often the additional samples turned out to be the best design thereby improving the quality of the surrogate, and the quality of final solution is also improved. Since evaluation of the additional sampling points is done concurrently, the total CPU time to solve the problem is significantly decreased.

As demonstrated by the simple case study analyzed herein, scheduling of polymer injection is in general non trivial and application of optimization techniques is advisable. It was observed that the optimal solution started out injecting water on the first two years and after that polymer injection began. For this example it can be concluded that polymer should start at the first years of the concession period rather than at the end. It was also observed that optimal solution is sensitive to oil price, with the method becoming increasingly feasible with higher oil prices.

**Nomenclature**

**Acronyms**

BHP	bottom hole pressure
BLUP	best linear unbiased predictor
DACE	Design and Analysis of Computer Experiments
DOE	design of experiments
EGO	efficient global optimization
EOR	enhanced oil recovery
ISC	infill sampling criterion

IPV	inaccessible pore volume constant
PRESS	prediction residual error sum of square
RMSE	root mean squared error
RRF	residual resistance factor

**Symbols**

$A_{dmax}$	maximum adsorptive capacity of the rock
CWI	cumulative water injection
COP	cumulative oil production
Corr	correlation term
$C_p$	concession period
CPI	cumulative polymer injection
$d(\mathbf{x}^{(i)}, \mathbf{x}^{(j)})$	normalized distance
$EI(x)$	expected improvement function
$f$	true function
$\hat{f}$	approximate function
$\mathbf{f}$	true functions at samples
$I(x)$	the improvement function
$lf$	likelihood function
$m$	total number of samples in a DOE scheme
$niw$	number of injector wells
$n$	total number of design variables
$n_d$	number of promising design for the parallel ISC scheme
$n_p$	number of computer processors
$N_j$	regressors
NPVRI	alternative objective function
OF	objective function
$op$	oil price
$pc$	polymer cost
$\mathbf{R}$	regression matrix
RI	relative improvement of injection schedule (objective function)
$S_o$	initial oil saturation
$S^2$	mean squared error of the predictor
TOL, tol	prescribed tolerances
wic	water injection cost
$\mathbf{x}$	design variables

**Greek symbols**

$\beta_j, \theta_j$	Kriging unknown parameters
$\varepsilon$	error in the Kriging model
$\varepsilon_M$	machine epsilon
$\varphi$	standard normal density function
$\phi$	reservoir rock porosity
$\bar{\phi}$	polymer affected porosity

$\Phi$	standard normal cumulative distribution function
$\mu$	mean of the stochastic process
$\sigma$	standard deviation

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