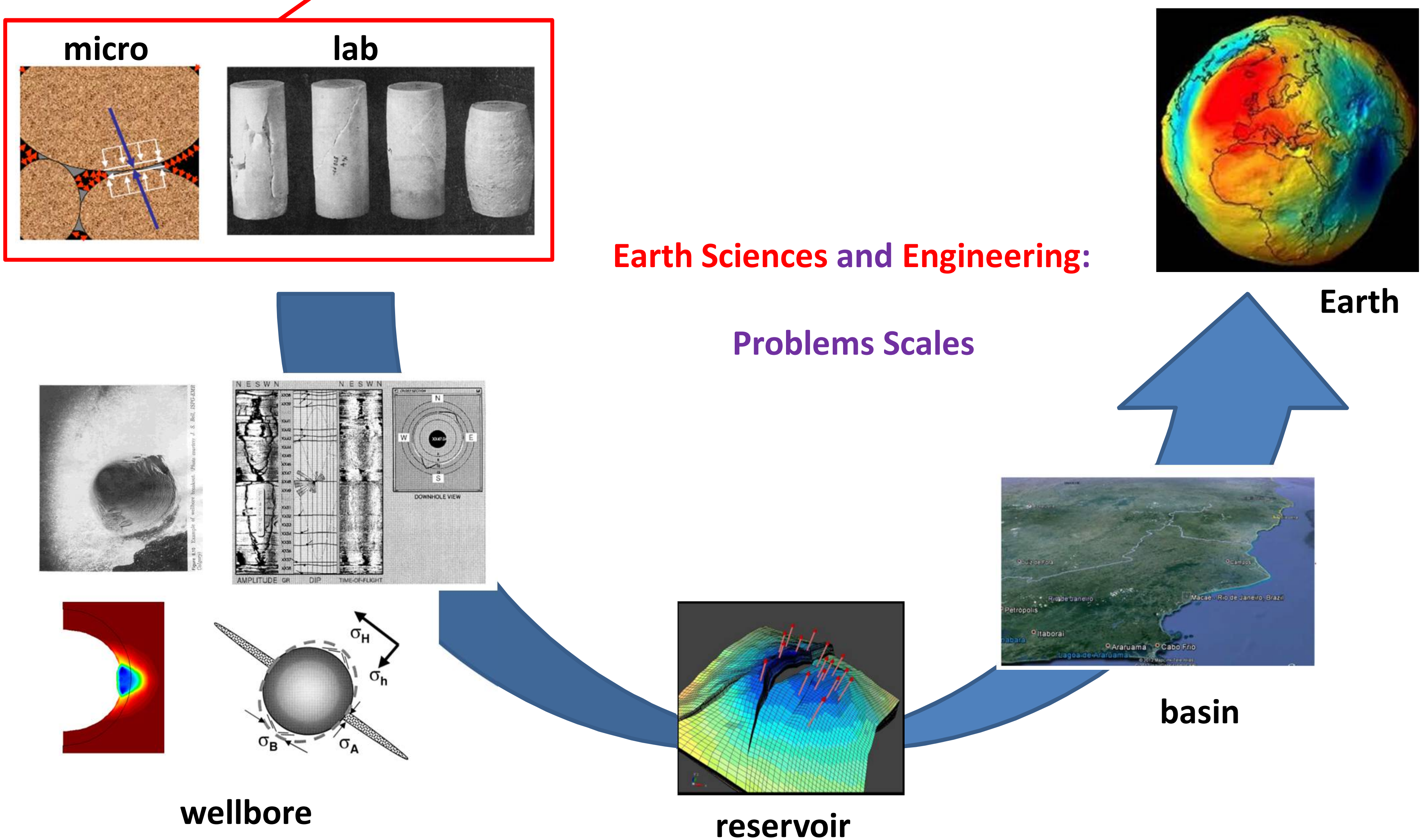
Introduction to Reservoir Geomechanics

UFPE

UFPE

Constitutive Laws



Deformable Porous Media

- **Stresses**

✓ Continuum mechanics deals with deformable bodies.

✓ The stresses considered in continuum mechanics are only those produced by deformation of the body.

✓ Stress is a measure of the average force per unit area of a surface within a deformable body on which internal forces act.

✓ It is a measure of the intensity of the internal forces acting between particles of a deformable body across imaginary internal surfaces.

✓ These internal forces are produced between the particles in the body as a reaction to external forces applied on the body.

✓In general, stress is not uniformly distributed over the cross-section of a material body, and consequently the stress at a point in a given region is different from the average stress over the entire area.

✓Therefore, it is necessary to define the stress not over a given area but at a specific point in the body.

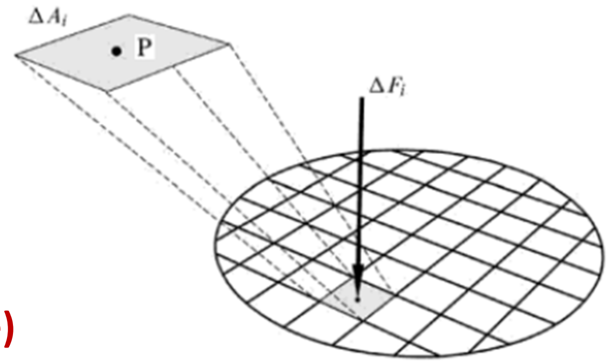
Deformable Porous Media

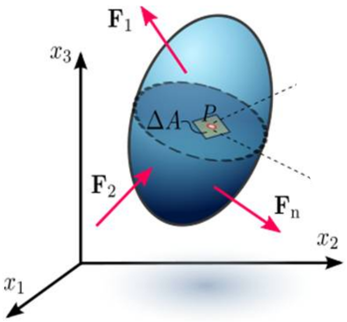
Normal and Shear Stresses:

Definition:

$$\sigma = \lim_{\Delta A_i \to \infty} \frac{\Delta F_i}{\Delta A_i}$$

stress vector
(acting in a infinitesimal surface)

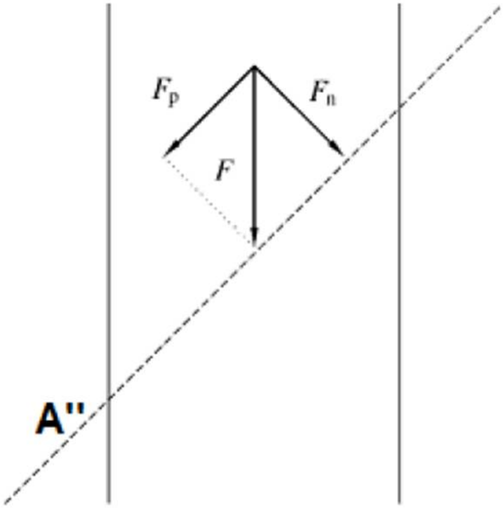


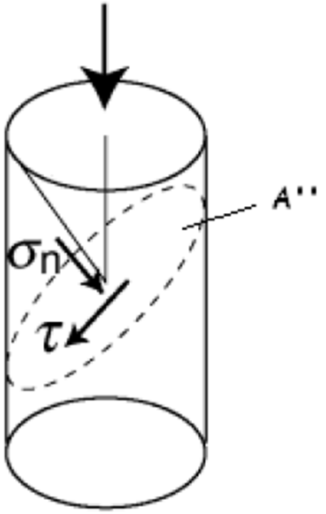


Decomposition of stress vector:

Normal stress: $$\sigma = \frac{F_n}{A''}$$

Shear stress: $$\tau = \frac{F_p}{A''}$$

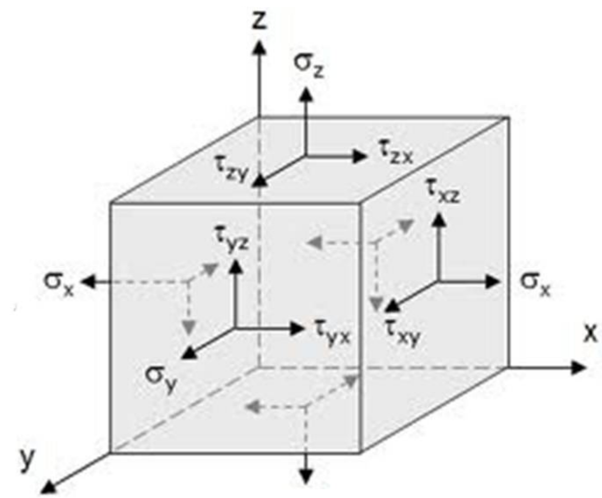


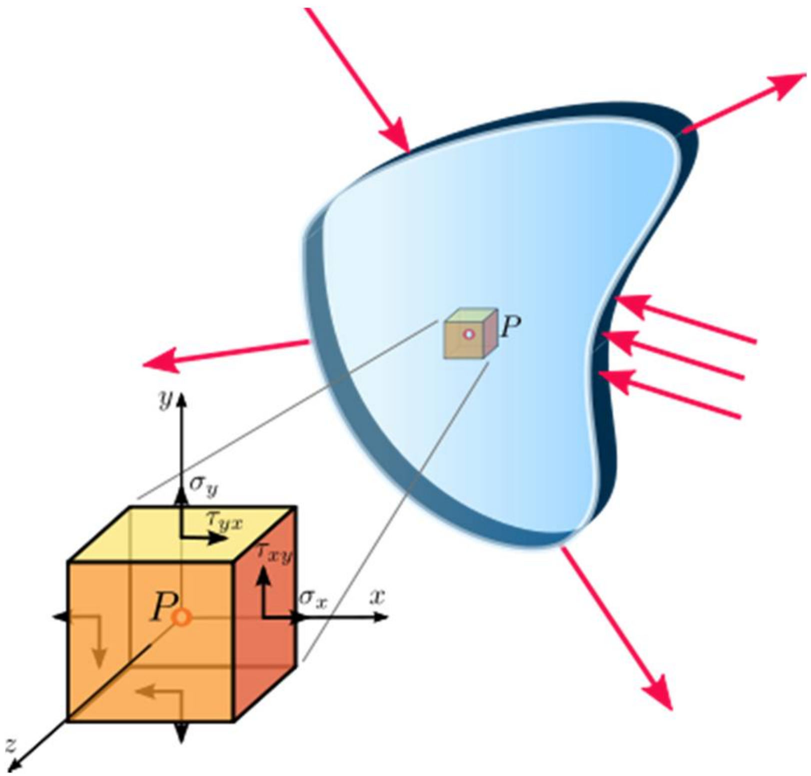


Stress Representation

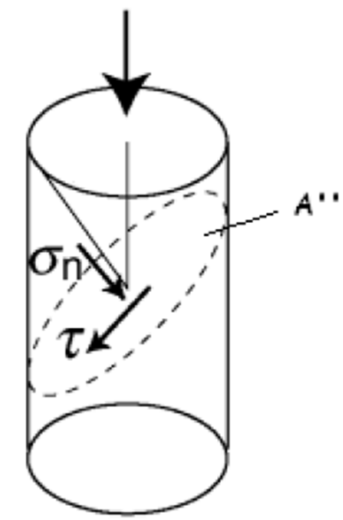
In 3 dimensions :

$$s = \begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix} = \begin{vmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{vmatrix} \begin{Vmatrix} \mathbf{i} \\ \mathbf{j} \\ \mathbf{k} \end{Vmatrix}$$





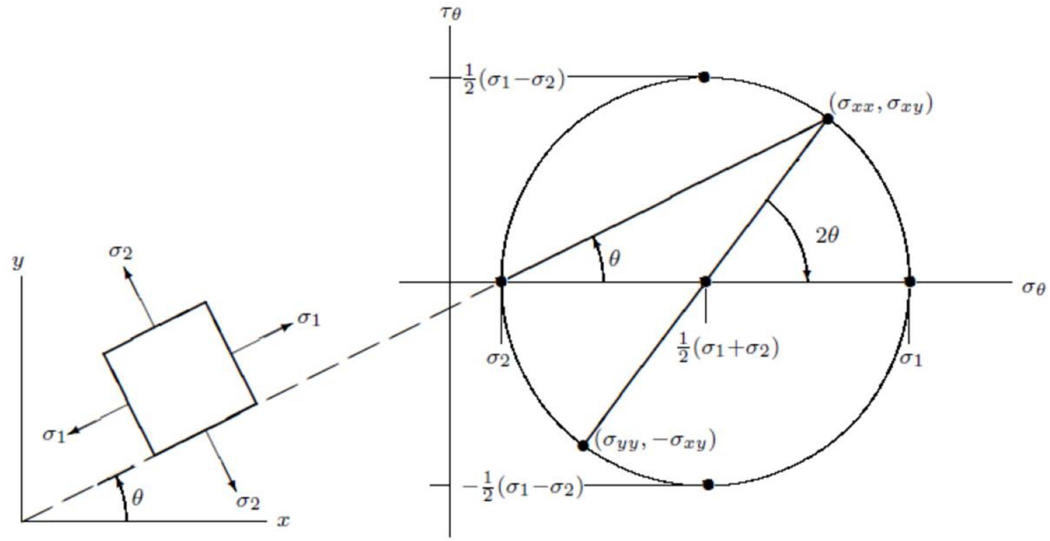
Defined for each point for a body in equilibrium, subjected to external load



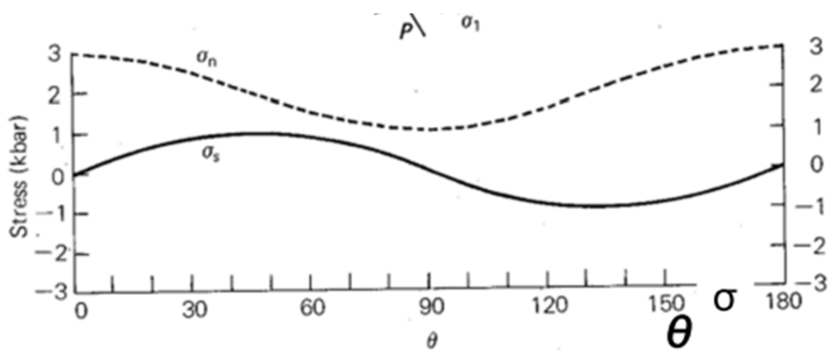
Stress Representation

Mohr Circle:

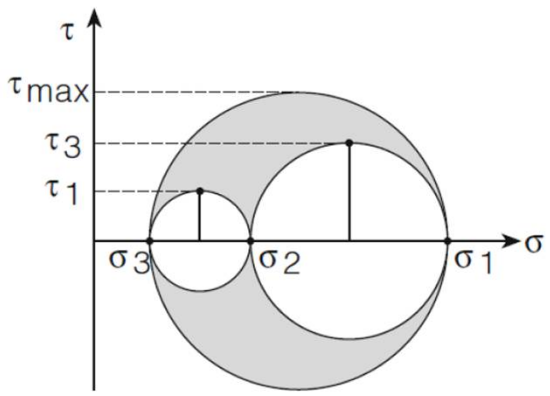
$$\begin{vmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{vmatrix}$$



Extract from the Mohr circle changes of normal and shear stress with orientation



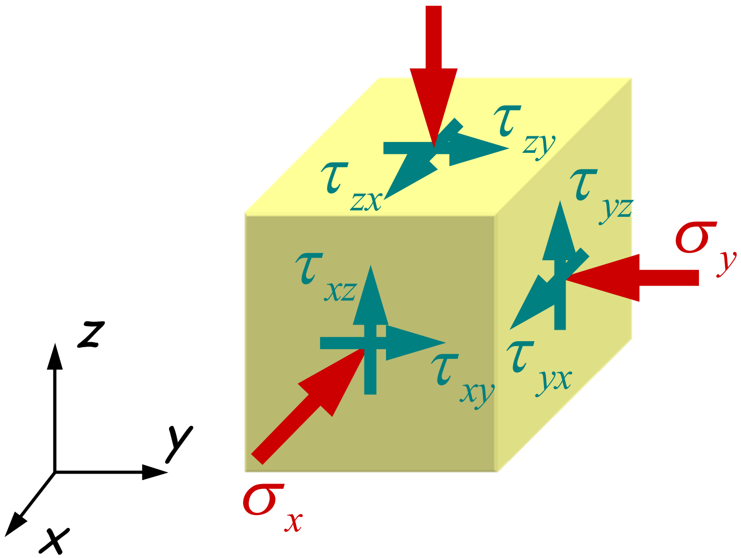
3D representation of Mohr Circle:



Deformable Porous Media

➢ Cauchy Stresses

✓ According to Cauchy, the stress at any point in an object assumed to behave as a continuum is completely defined by the nine components of a second-order tensor of type known as the Cauchy stress tensor.



$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

e.g. τ_{xy} acts on the *x* face in the *y* direction,

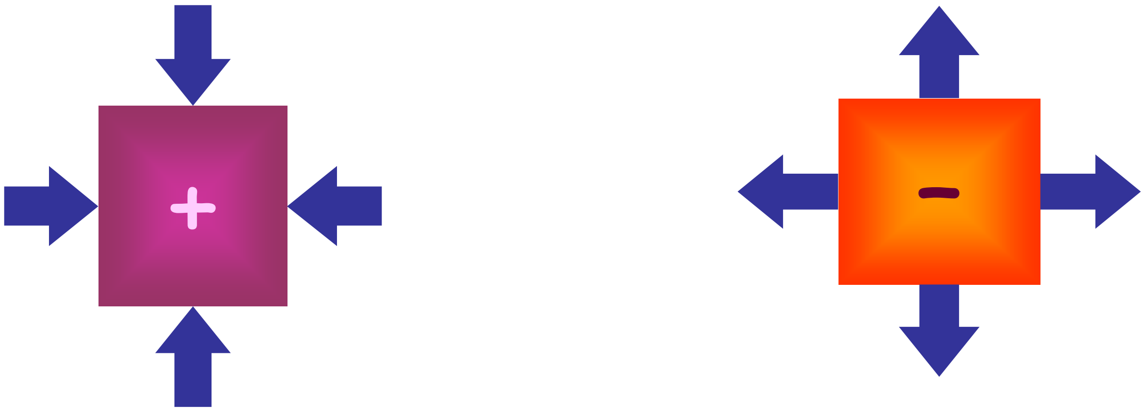
✓ The Cauchy stress tensor is used for stress analysis of material bodies experiencing small deformations.

✓For large deformations, also called finite deformations, other measures of stress, such as the first and second Piola-Kirchhoff stress tensors, the Biot stress tensor, and the Kirchhoff stress tensor, are required

Deformable Porous Media

✓ Stress Sign Convention

In Soil/Rock Mechanics compression is considered as positive and tension as negative.

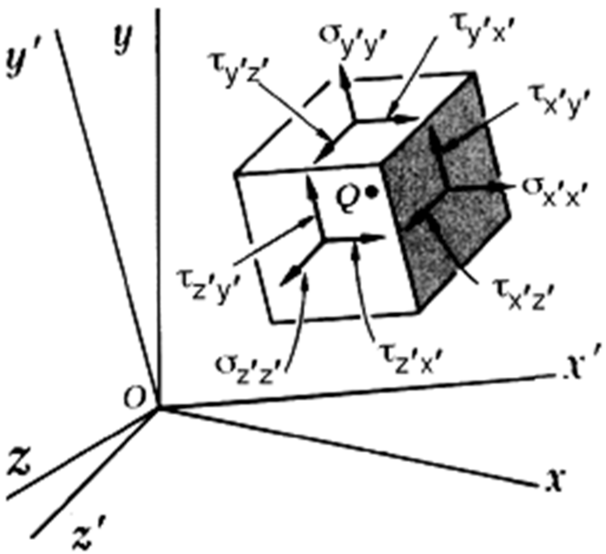


Compressive stresses are positives

Tensile stresses are negatives

Stress, stress changes in space and time

In 3 dimensions :



$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}_{xyz}$$

full tensor

Rotation of reference axes

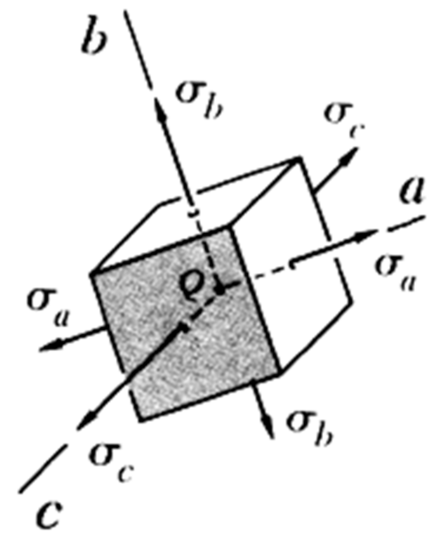
Among the infinite number of triplets of planes which satisfy the fundamental stress Theorem, there is always one set (=abc) on which no shear stress is present.

Principal stresses:

$$\sigma_1 = \max(\sigma_a, \sigma_b, \sigma_c)$$

$$\sigma_3 = \min(\sigma_a, \sigma_b, \sigma_c)$$

$$\sigma_2 = \text{the remaining component}$$



$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}_{abc}$$

diagonal tensor

Stress, stress changes in space and time

➢ Principal Stresses

✓ **The Cauchy stress tensor obeys the tensor transformation law under a change in the system of coordinates.**

✓**A graphical representation of this transformation law is the Mohr's circle for stress**

✓ **At every point in a stressed body there are at least three planes, called principal planes, with normal vectors , called principal directions, where the corresponding stress vector is perpendicular to the plane and where there are no normal shear stresses.**

✓**The three stresses normal to these principal planes are called principal stresses**

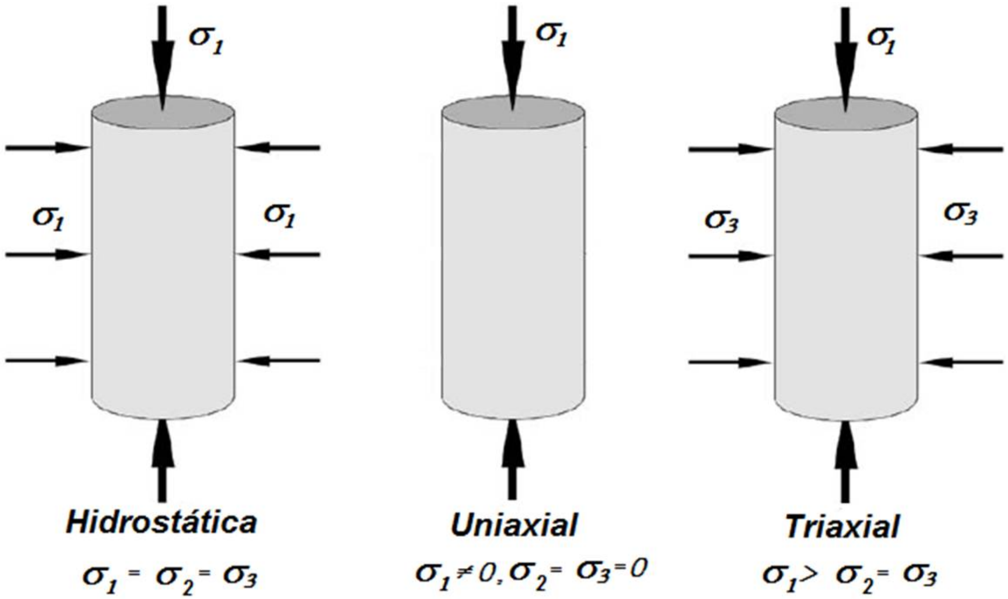
$\sigma_1 \geq \sigma_2 \geq \sigma_3$ ⇢ Principal stresses

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

Stress Representation

Stress state in lab experiments:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_1 & 0 \\ 0 & 0 & \sigma_1 \end{bmatrix}$$



$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_3 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

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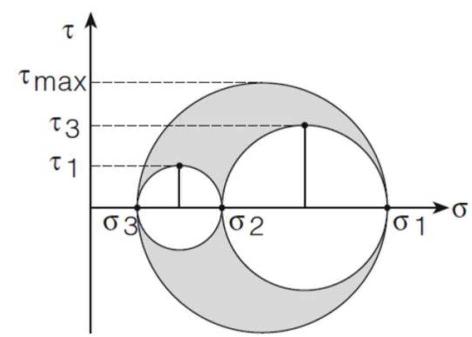
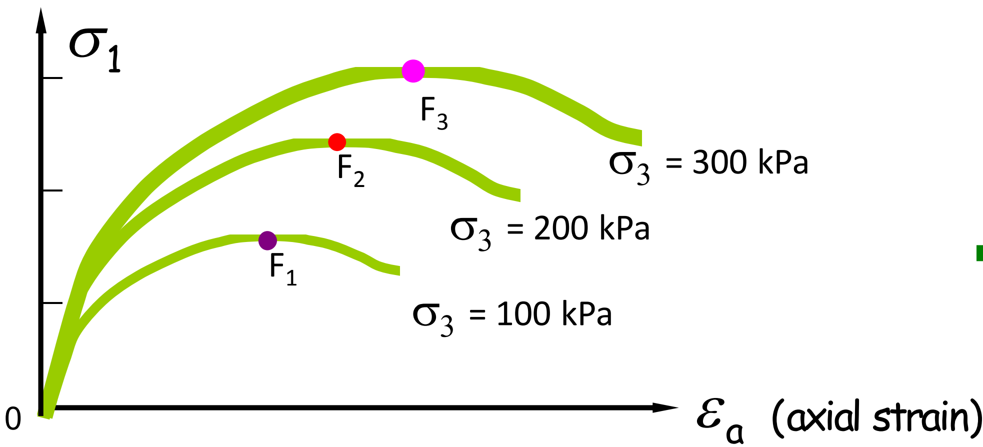
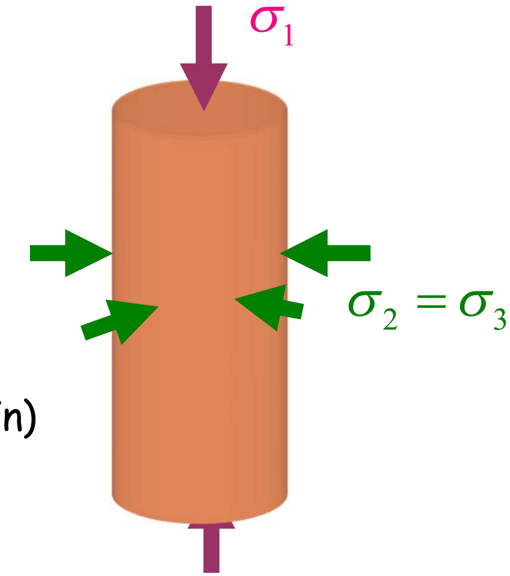
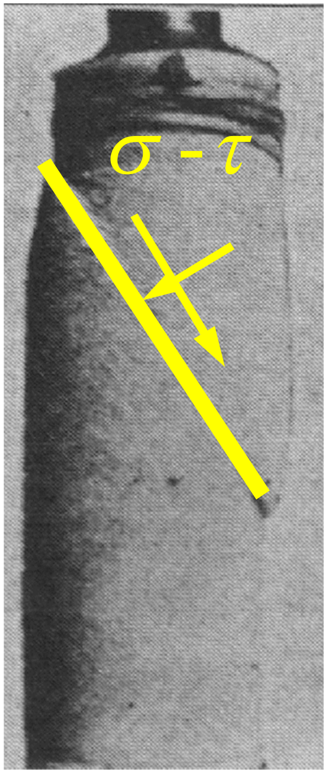


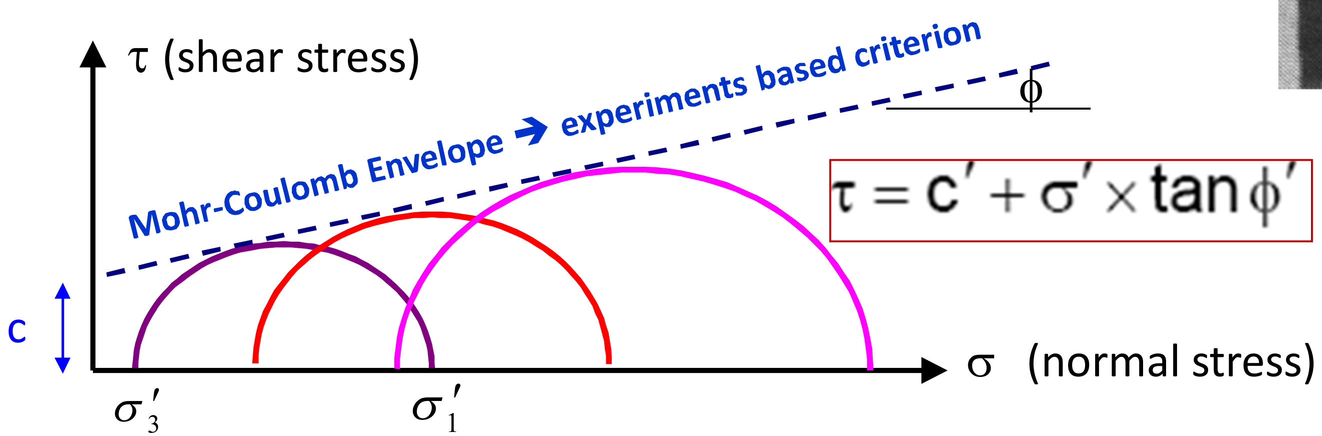
TRIAXIAL TEST







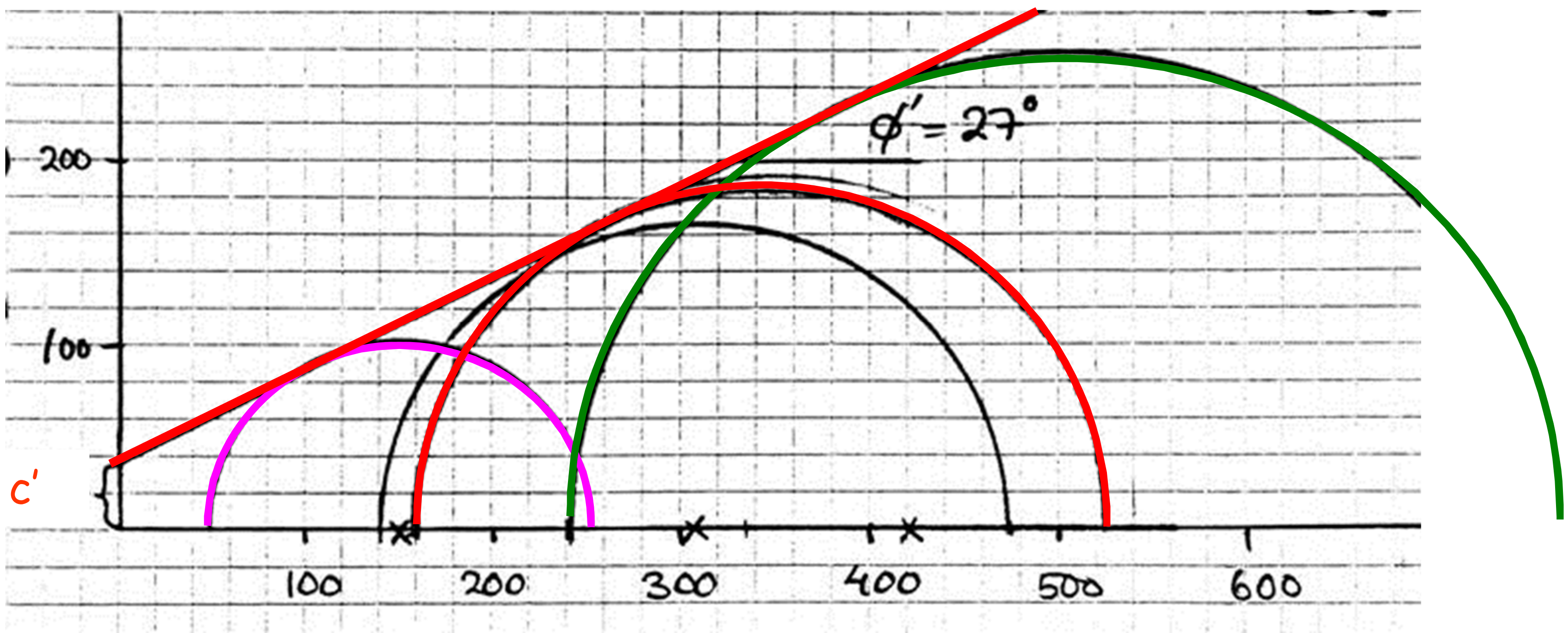
Mohr-Coulomb criteria for shear strength (minimum 3 samples):



$$\tau = c' + \sigma' \times \tan\phi'$$

TRIAXIAL TEST

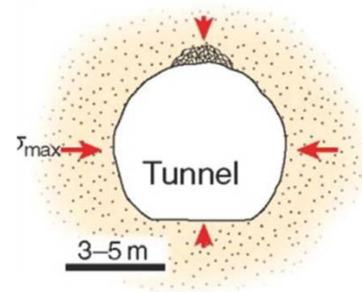
Test	σ'_3 (kPa)	$(\sigma'_1 - \sigma'_3)$ (kPa)	σ'_1 (kPa)	$1/2(\sigma'_1 - \sigma'_3)$ (kPa)	$1/2(\sigma'_1 + \sigma'_3)$ (kPa)
1	50	200	250	100	150
2	140	335	475	168	308
3	300	520	760	260	500

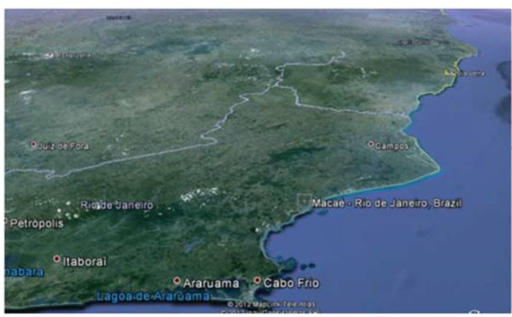


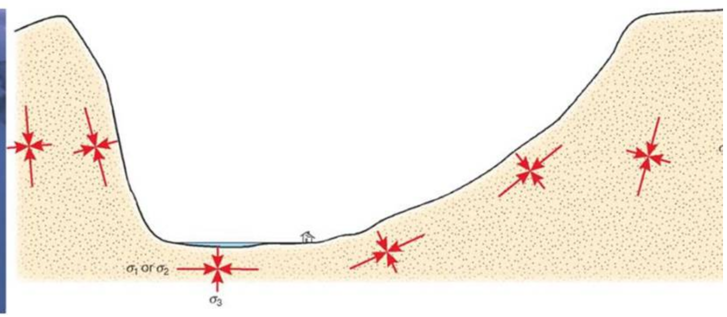
Stress Fields

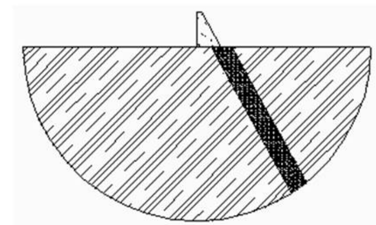
Principal stresses and principal planes of stress:

Very useful because
i) the position of principal surfaces of stress can often be identified
ii) the orientation of structures (faults etc) depends on the position of the principal stresses

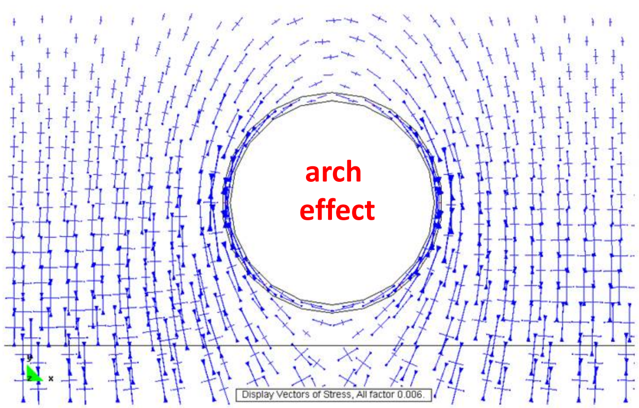


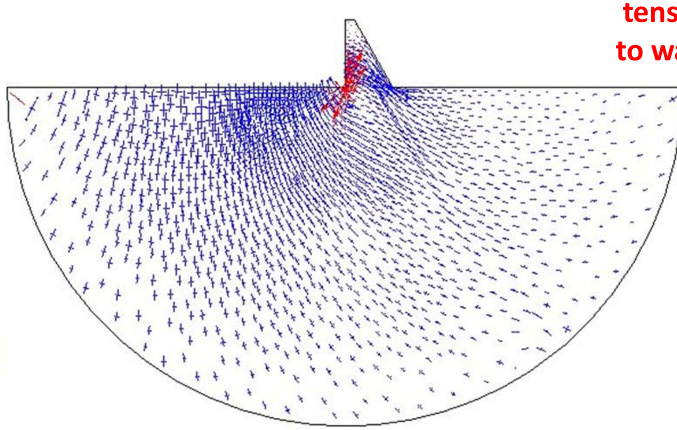


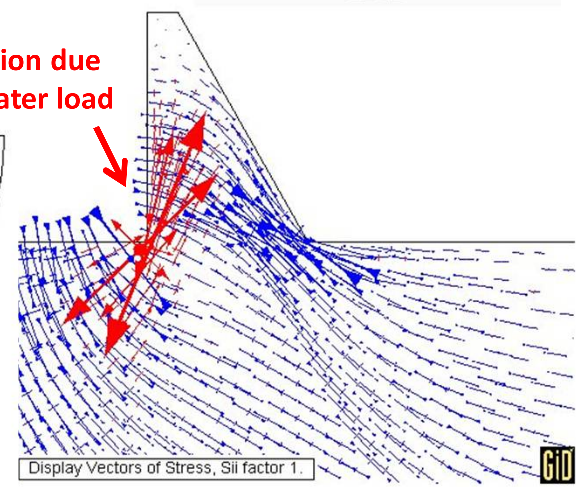




Where are surfaces of principal stresses?



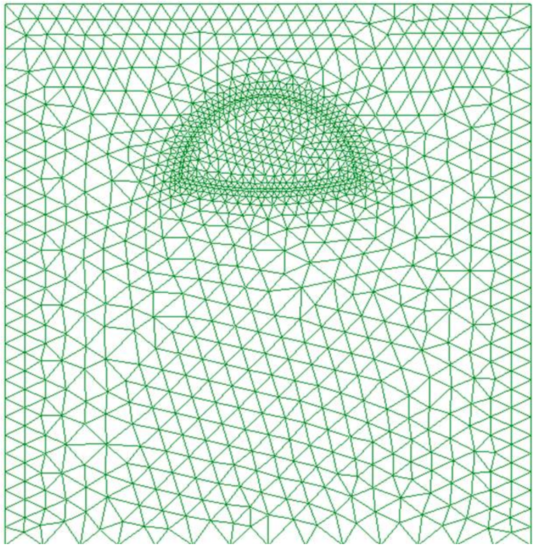


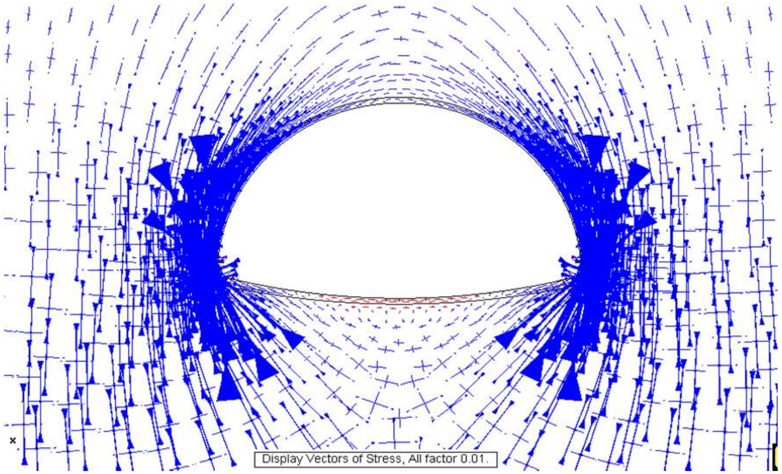


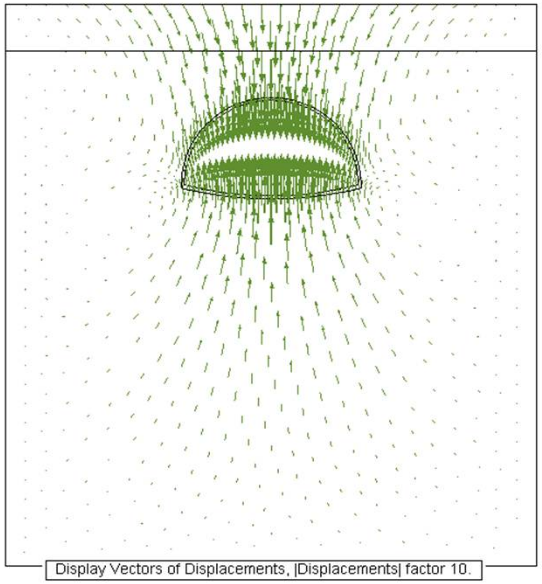
Stress Fields

In 2D analysis: stress field and displacement field:

Finite element analysis of a tunnel excavation:







Mesh

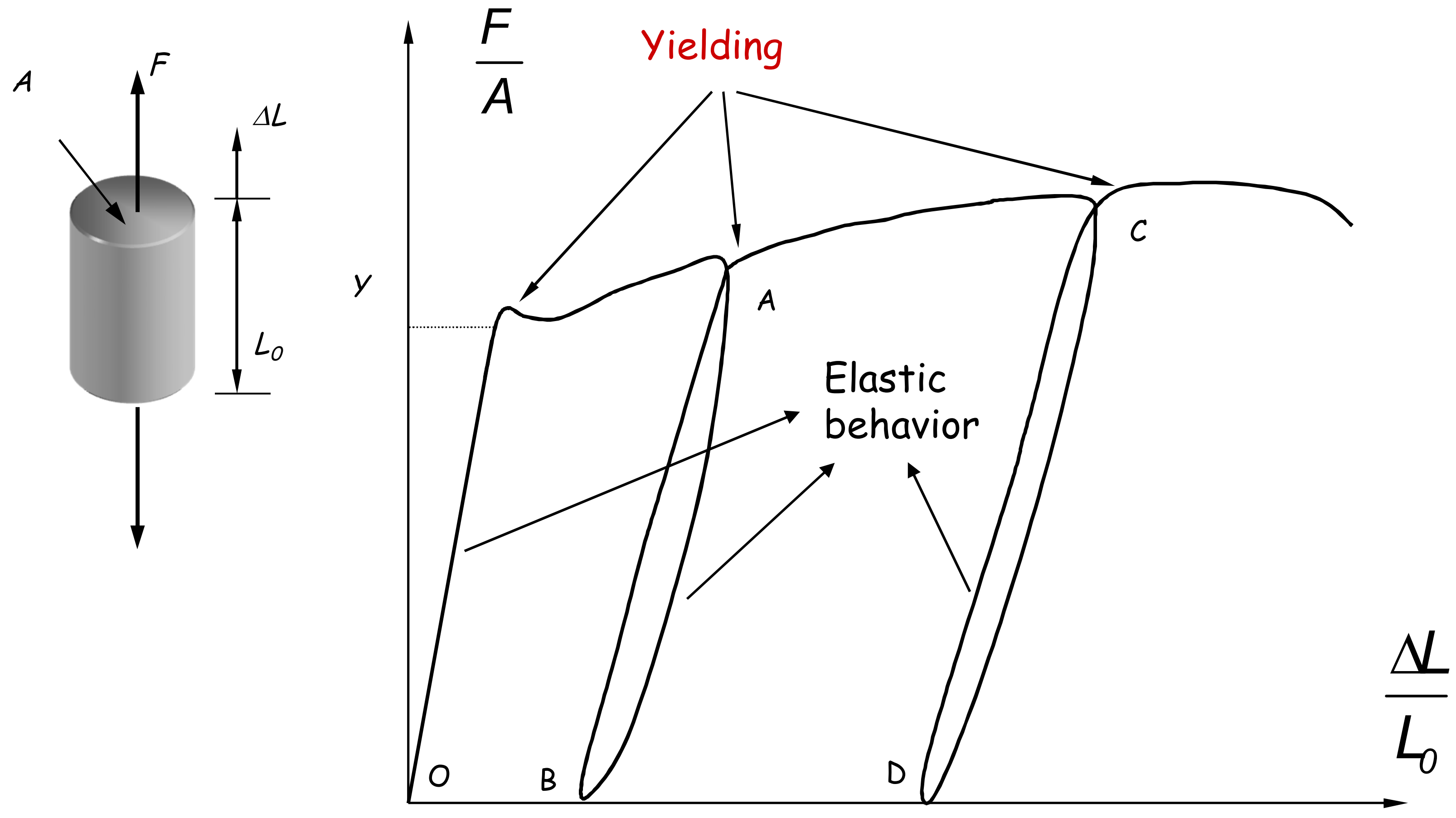
Stress field

Displacement field

Stresses are related to Strains

Axial behavior

- ✓ Example of elasto-plastic behaviour: tensile test (1D) in metals



Stresses are related to Strains

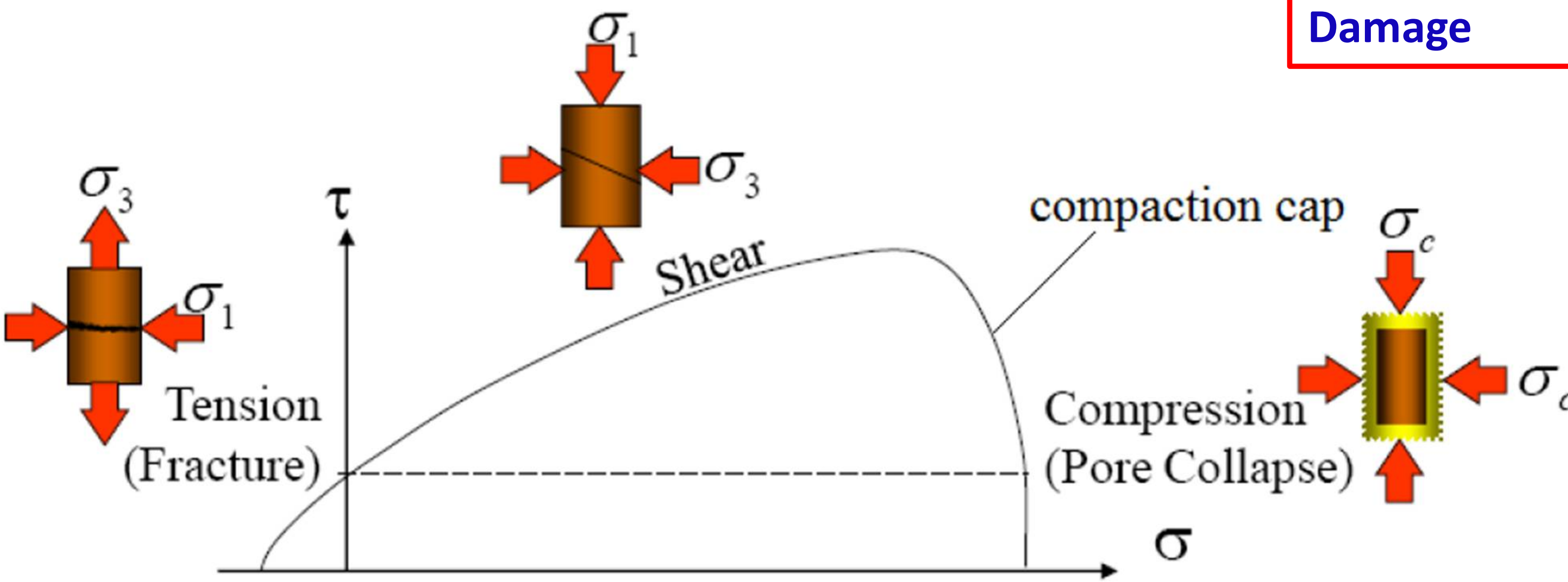
- **3D behavior**

$$\boldsymbol{\sigma}' = \mathbf{D} \cdot \boldsymbol{\varepsilon}$$ → **stress-strain relationship**

D: constitutive tensor →

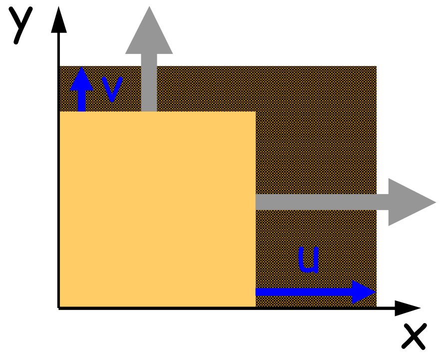
- **Elasticity**
- **Visco-Elasticity**
- **Plasticity**
- **Visco-Plasticity**
- **Damage**

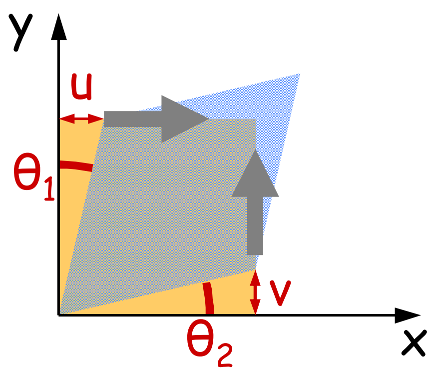
CAP Model: Elastoplastic multi-mechanism model



Strains are related to Displacements

❑ Strain Tensor





Cauchy's infinitesimal strain tensor:

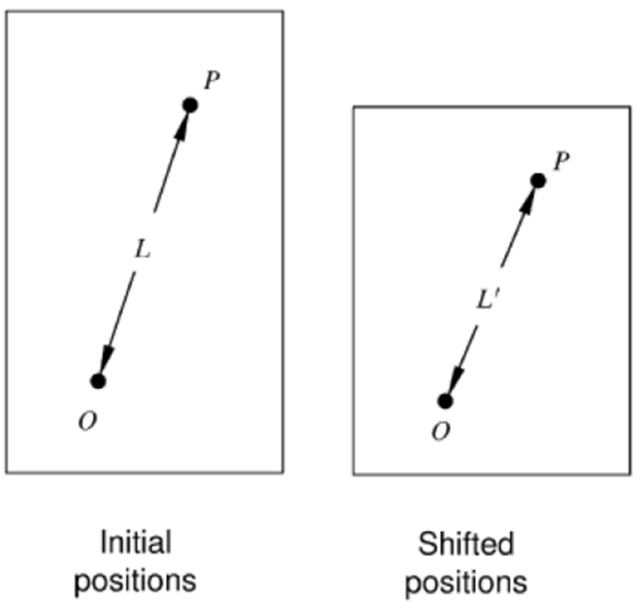
$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_z \end{bmatrix}$$

Compatibility conditions:

$$\boldsymbol{\varepsilon} = \frac{1}{2}\left(\nabla \mathbf{u} + \nabla \mathbf{u}^T\right)$$

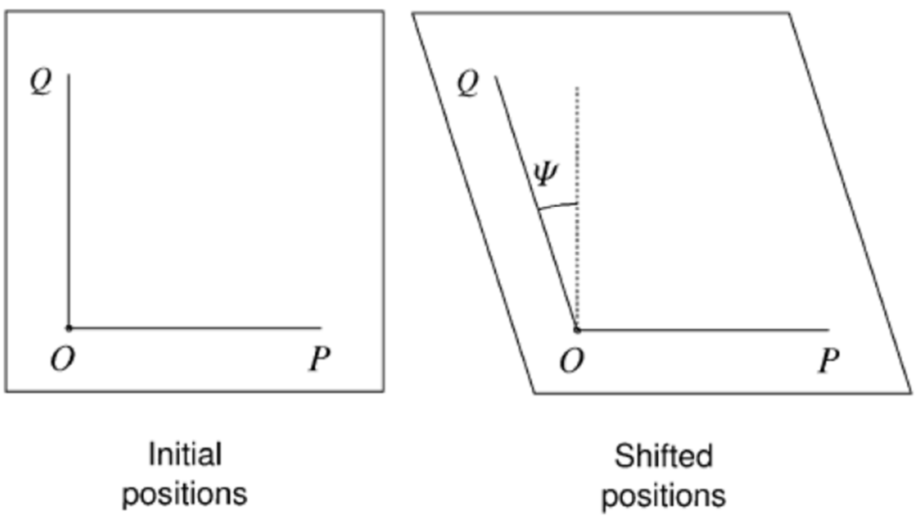
Strains

extension/contraction:



$$\varepsilon = \frac{L - L'}{L} = -\frac{\Delta L}{L}$$

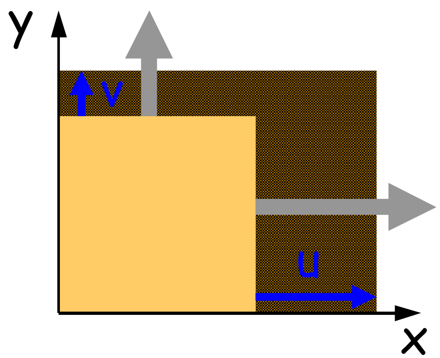
distortion:

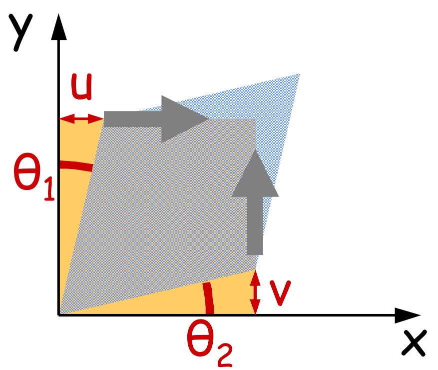


$$\Gamma = \frac{1}{2} tan \Psi$$

Strains are related to Displacements

❑ Strain Tensor





- ✓ Compression +
- ✓ Small deformations :

$$\boldsymbol{\varepsilon} = \frac{1}{2}\left(\nabla\mathbf{u} + \nabla\mathbf{u}^T\right)$$

Symmetric part of the displacement gradient tensor

Displacement vector
$\mathbf{u} = (u,v,w)^T$

Component x: u
Component y: v
Component z: w

or:

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \varepsilon_y = \frac{\partial v}{\partial y} \qquad \varepsilon_z = \frac{\partial w}{\partial z}$$

+

$$\gamma_{xy} = \theta_{xy} + \theta_{yx} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = \varepsilon_{xy} + \varepsilon_{yx} = 2\varepsilon_{xy}$$

$$\gamma_{xz} = \theta_{xz} + \theta_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = \varepsilon_{xz} + \varepsilon_{zx} = 2\varepsilon_{xz}$$

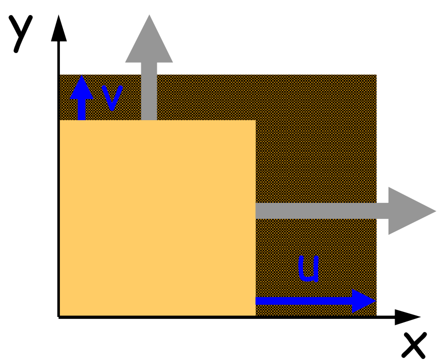
$$\gamma_{yz} = \theta_{yz} + \theta_{zy} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = \varepsilon_{yz} + \varepsilon_{zy} = 2\varepsilon_{yz}$$

Strains are related to Displacements

❑ Strain Tensor

✓ Compression +

✓ Small deformations :



$$\boldsymbol{\varepsilon} = \frac{1}{2}\left(\nabla\mathbf{u} + \nabla\mathbf{u}^T\right)$$

Symmetric part of the displacement gradient tensor

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or:

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \varepsilon_y = \frac{\partial v}{\partial y} \qquad \varepsilon_z = \frac{\partial w}{\partial z}$$

volumetric strain:

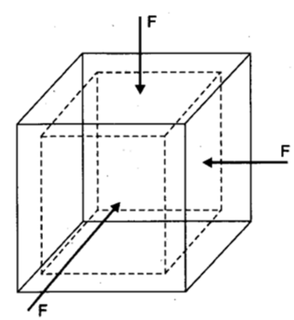
$$\varepsilon_v = \frac{\delta V}{V_0} = \nabla \cdot \mathbf{u} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

Divergence of the displacement vector

Note:

Stress Sign Convention:

if **positive** for **compression**



$$\boldsymbol{\varepsilon} = -\frac{1}{2}\left(\nabla\mathbf{u} + \nabla\mathbf{u}^T\right)$$

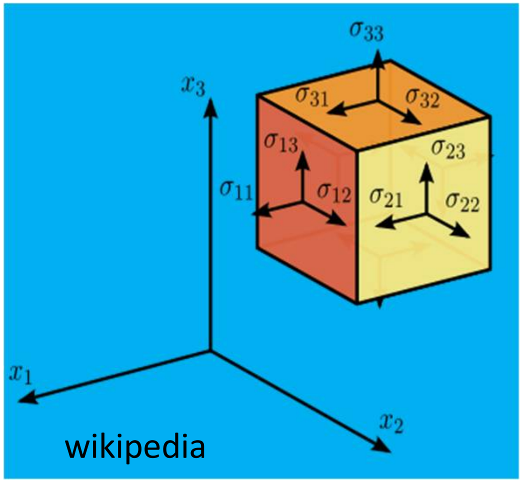
$$\varepsilon_v = -\frac{\delta V}{V_0} = -\nabla \cdot \mathbf{u} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

Equilibrium Equation

UFPE

MOMENTUM BALANCE (EQUILIBRIUM)

- Consider a volume of a porous medium.
- If **inertial terms** are **neglected**, the distribution of **total stress** within this volume can be shown to satisfy the equilibrium equation.



$$\frac{\partial \sigma_{ij}}{\partial x_j} + F_i = 0 \qquad i = 1, 2, 3$$

or

$$\nabla \cdot \boldsymbol{\sigma} + b = 0$$

- Where the σ_{ij} (or σ) is the **total stress** on the medium and F_i (or b) is the body force per unit volume of the medium.
- The nine stress components are shown in figure above.

Equilibrium Equation

MOMENTUM BALANCE (EQUILIBRIUM)

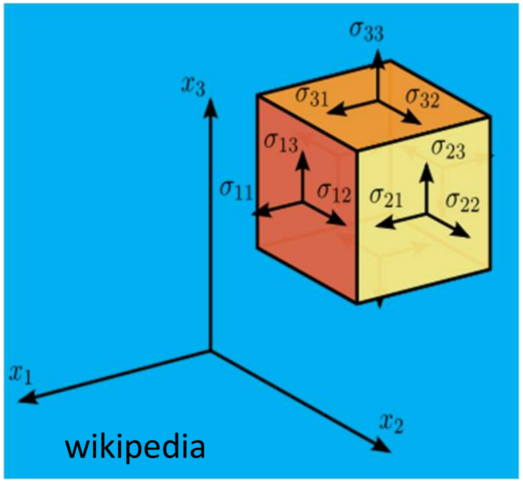
➢ Consider a volume of a porous medium.

➢ If **inertial terms** are **neglected**, the distribution of **total stress** within this volume can be shown to satisfy the equilibrium equation.

$$\frac{\partial \sigma_{ij}}{\partial x_j} + F_i = 0 \qquad i = 1, 2, 3$$

or

$$\nabla \cdot \underbrace{(\boldsymbol{\sigma}' + \alpha \cdot p_f \cdot \mathbf{I})}_{\boldsymbol{\sigma}} + b = 0$$



➢ Where the σ_{ij} (or σ) is the **total stress** on the medium and F_i (or b) is the body force per unit volume of the medium.

➢ The nine stress components are shown in figure above.

Equilibrium Equation

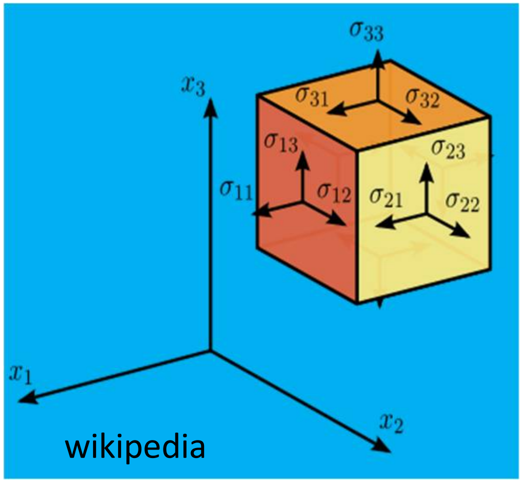
MOMENTUM BALANCE (EQUILIBRIUM)

- Consider a volume of a porous medium.
- If **inertial terms** are **neglected**, the distribution of **total stress** within this volume can be shown to satisfy the equilibrium equation.

$$\frac{\partial \sigma_{ij}}{\partial x_j} + F_i = 0 \qquad i = 1, 2, 3$$

or

$$\nabla \cdot (\underbrace{\overbrace{\mathbf{D} \cdot \boldsymbol{\varepsilon}}^{\boldsymbol{\sigma}'} + \alpha \cdot p_f \cdot \mathbf{I}}_{\boldsymbol{\sigma}}) + b = 0$$



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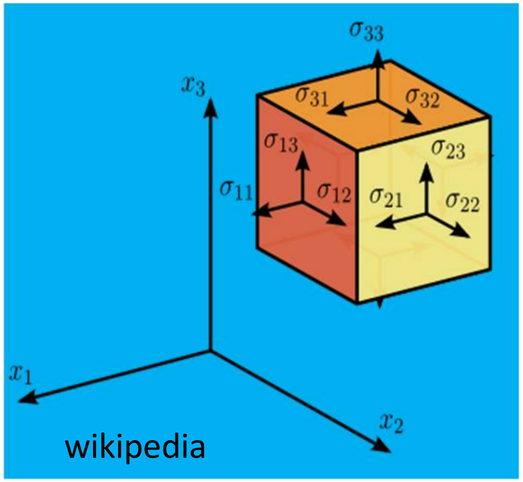
Equilibrium Equation

MOMENTUM BALANCE (EQUILIBRIUM)

- Consider a volume of a porous medium.
- If **inertial terms** are **neglected**, the distribution of **total stress** within this volume can be shown to satisfy the equilibrium equation.

$$\underbrace{\nabla \cdot \mathbf{D} \cdot \left[\left(\overbrace{\frac{1}{2}\left(\nabla \mathbf{u} + \nabla \mathbf{u}^{T}\right)}^{\boldsymbol{\varepsilon}} \right) + \alpha \cdot p_{f} \cdot \mathbf{I} \right]}_{\boldsymbol{\sigma}} + b = 0$$



- Where the σ_{ij} (or σ) is the **total stress** on the medium and F_i (or b) is the body force per unit volume of the medium.
- The nine stress components are shown in figure above.

Effective Stress Principle

- Pore Water Pressure, Total & Effective Stresses
- Effective Stress

Karl Terzaghi
(1883 - 1963)

- ✓ Terzaghi (1936) proposed the principle of effective stress(*), the most important equation in soils mechanics.
- ✓ The effective stress (σ') is the component of the normal stress taken by the soil skeleton.
- ✓ It is the effective stress which controls the volume and the strength of the soil.
- ✓ It is assumed saturated soil, water incompressibility and rigid soil particles.

$$\sigma' = \sigma - u_w$$

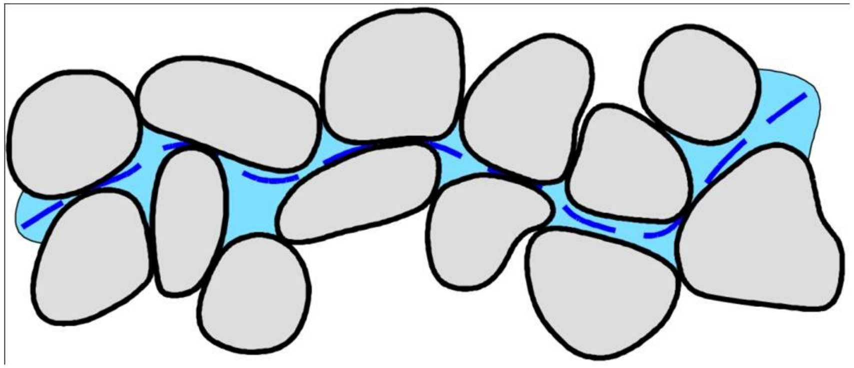
efective stress total stress pore water pressure

(*) *"All the measurable effects of a change of stress, such as compression, distortion and a change in the shearing resistance are exclusively due to changes in effective stress…every investigation of the stability of a saturated body of earth requires the knowledge of both the total and the neutral stresses."* (Terzaghi, 1936)

Effective Stress Principle

➢ Normal Stress in the Stress Tensor:

$$\sigma' = \sigma - p_w$$



- Multiphase material (solid and liquid)
- Incorporation of an additional variable: pore pressure
- Coupled phenomena (mechanical & hydraulic)

Effective Stress Principle

The concept of effective stress is based on the pioneering work in soil mechanics by Terzaghi (1923) who noted that the behavior of a soil (or a saturated rock) will be controlled by the effective stresses, the differences between total stresses and pore pressure. The so-called "simple" or Terzaghi definition of effective stress is:

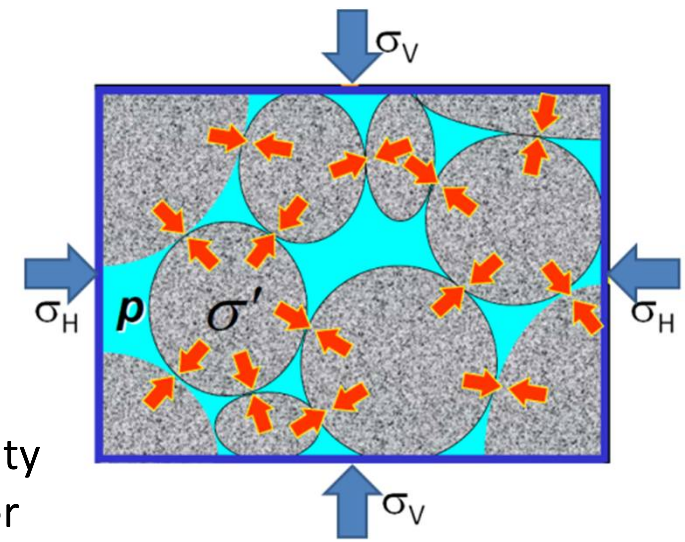
$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} - p_f \cdot \mathbf{I}$$

Effective Stress Tensor

Total Stress Tensor

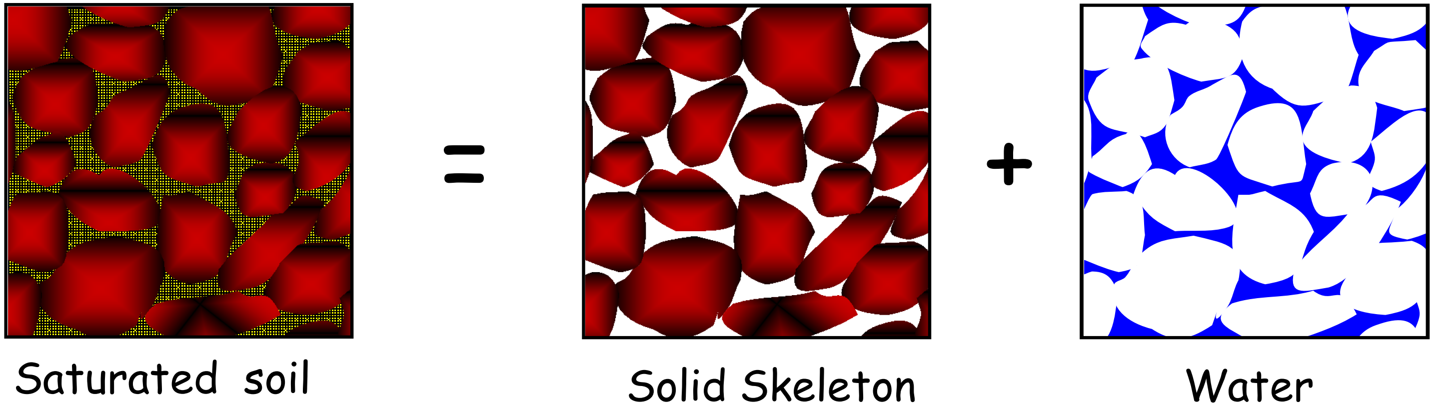
Fluid Pressure

Identity Tensor



Effective Stress Principle

A saturated porous medium comprises two phases:
- the soil particles
- the pore water



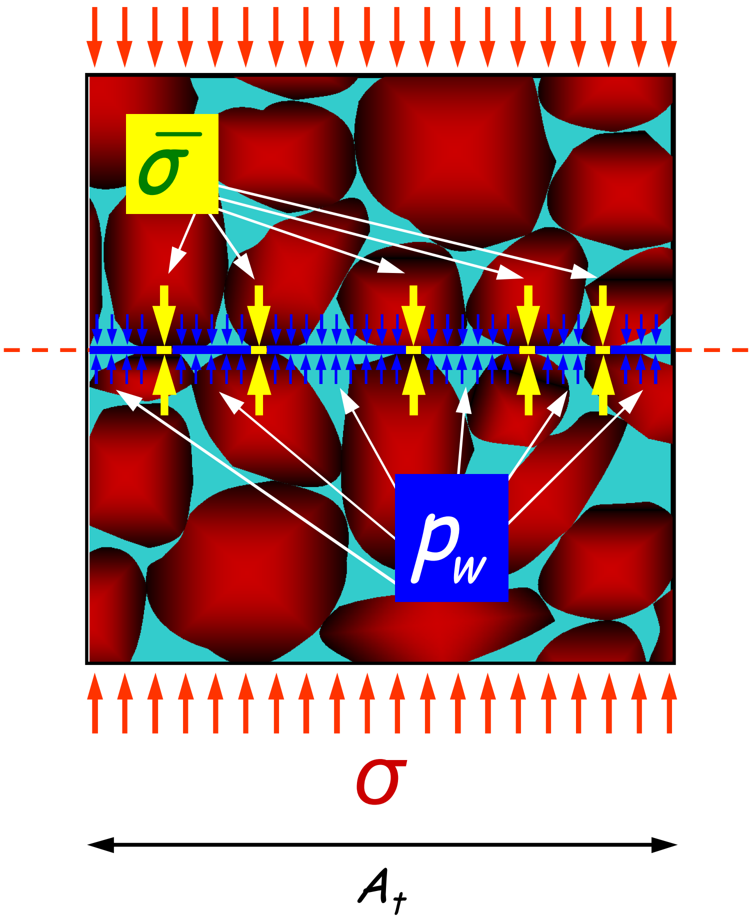
The strengths of these two phases are very different:

✓ the soil skeleton can resist shears. Two basic mechanisms:
- inter particle friction
- particles interlocking

✓ the shear strength of water is zero
- water can only sustains isotropic pressure.

Effective Stress Principle

✓Physical Interpretation



σ: total stresses externally applied

$\bar{\sigma}$: stresses that act through the contacts between particles (A_m)

p_w: water pressure (A_w)

A_t: total area

$$A_t = A_m + A_w$$

Effective Stress Principle

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} - p_f \cdot \mathbf{I}$$

In terms of normal components: $\sigma' = \sigma - p_f$

It is relatively straightforward to see that the stresses acting on individual grains result from the difference between the externally applied normal stresses and the internal fluid pressure. If one considers the force acting at a single grain contact, for example, all of the force acting on the grain is transmitted to the grain contact. Thus, the force balance is

$$F_T = F_g$$

which, in terms of stress and area, can be expressed as

$$\sigma \cdot A_T = \sigma_c \cdot A_c + (A_T - A_c)\, p_f$$

where A_c is the contact area of the grain and σ_c is the effective normal stress acting on the grain contact. Introducing the parameter $a = A_c/A_T$, this is written as

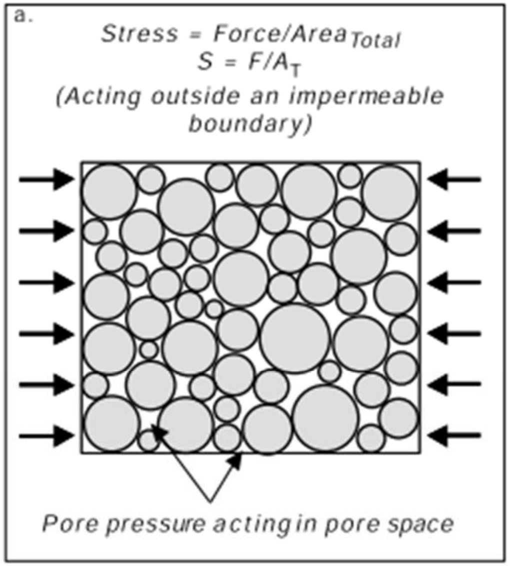
$$\sigma = a\sigma_c + (1-a)\, p_f$$

The intergranular stress can be obtained by taking the limit where a becomes vanishingly small

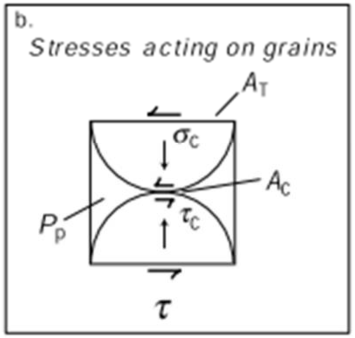
$$\lim_{a \to 0} a\sigma_c = \sigma'$$

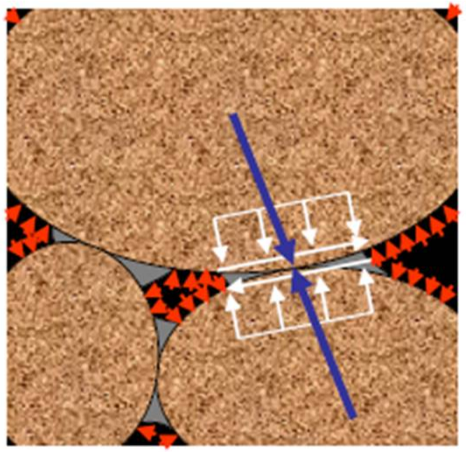
such that the "effective" stress acting on individual grains, σ' is given by

$$\sigma' = \sigma - (1-a)\, p_f = \sigma - p_f$$, for very small contact areas.



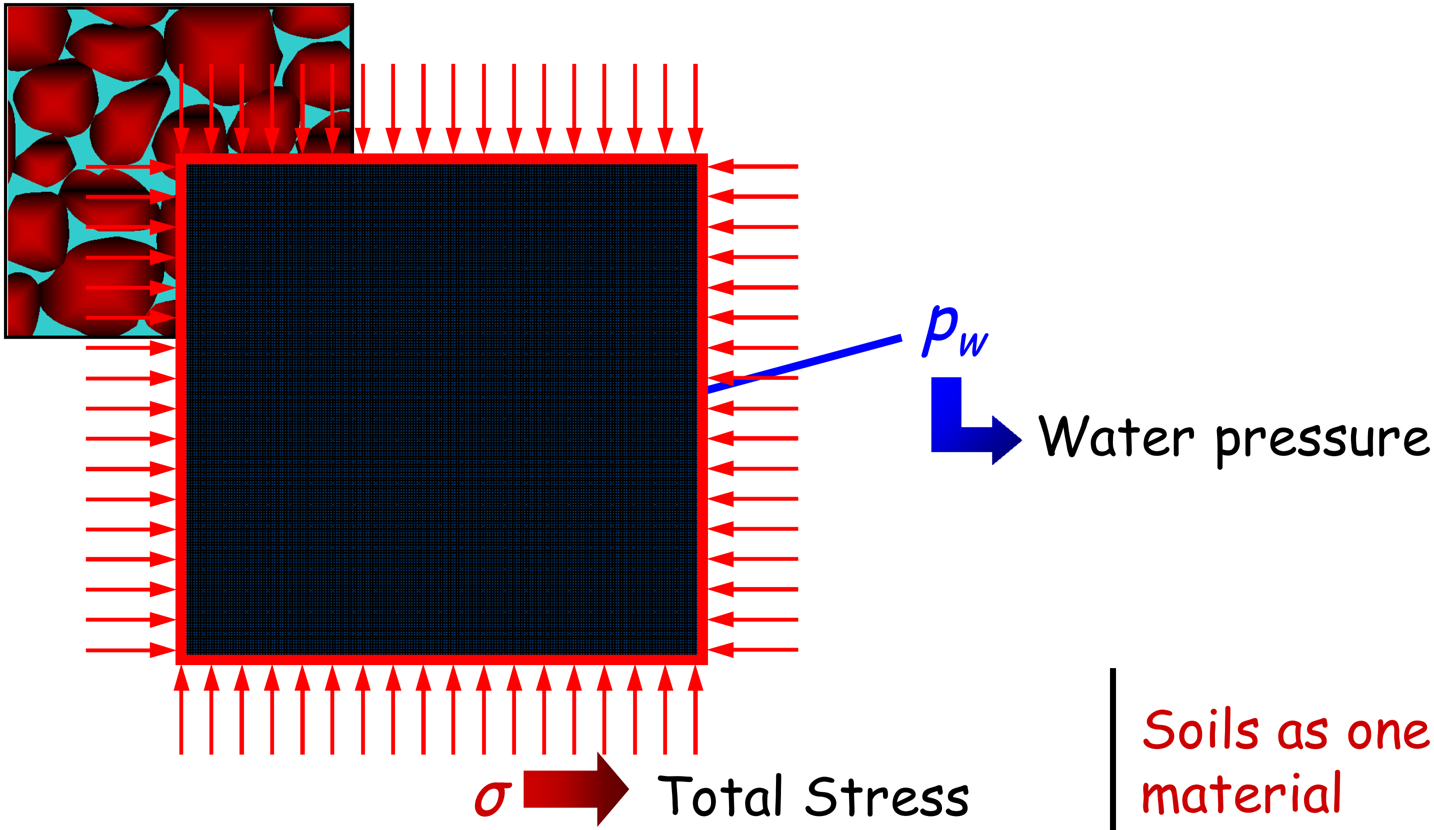
(Zoback, M. D., 2007)





Effective Stress Principle

The strength of the soil skeleton and the pore water are so different, therefore it is necessary to consider the stress acting in each phase separately



Soils as one phase material

Effective Stress Principle

UFPE

➢ Based on experimental information it can be concluded that Terzaghi's definition of effective stresses works well for a number of soils, but for other cases it needs an upgrade.

➢ A more general law for effective stresses can be expressed as:

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} - \alpha \cdot p_f \cdot \mathbf{I}$$

It is and extension of the one proposed by Terzaghi $(\sigma' = \sigma - p_w)$

where α is a physical constant known as Biot parameter.

➢ Geertsma (1957) and Skempton (1960) suggested:

$$\alpha = 1 - \left(\frac{K}{K_s}\right)$$

where:

K: is the drained bulk modulus of the dry aggregate or rock (i.e. porous medium skeleton).

K_s: is the bulk modulus of the soil's/rock's individual solid grains

Effective Stress Principle

- Based on experimental information it can be concluded that Terzaghi's definition of effective stresses works well for a number of soils, but for other cases it needs an upgrade.
- A more general law for effective stresses can be expressed as:

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} - \alpha \cdot p_f \cdot \mathbf{I}$$

$$\alpha = 1 - K / K_s$$

Biot's constant

K: Bulk modulus of the overall skeleton

K_s: Solid phase (rock grains) bulk modulus

For rocks, it is important to take into account the Biot's constant. $\alpha < 1$

For soils, it is equal to one. $\alpha = 1$

For unconsolidated or weak rocks, it is close to one. $\alpha \sim 1$

Effective Stress Principle

$$\alpha = 1 - K / K_s$$

- It is clear that: $0 \le \alpha \le 1$
- For solid rock (i.e. practically no interconnect pores): $K \approx K_s$

$$\lim_{\phi \to 0} \alpha = 0 \qquad \sigma_{ij} \approx \sigma'_{ij}$$

Therefore the pore pressure has no influence on porous media behavior.

- For a highly porous soil (e.g. soil with an open structure): $K <<< K_s$

$$\lim_{\phi \to 1} \alpha = 1 \qquad \sigma'_{ij} = \sigma_{ij} - p\,\delta_{ij}$$

Therefore the pore pressure has the maximum influence and the Terzaghi principle of effective stress is recovered.

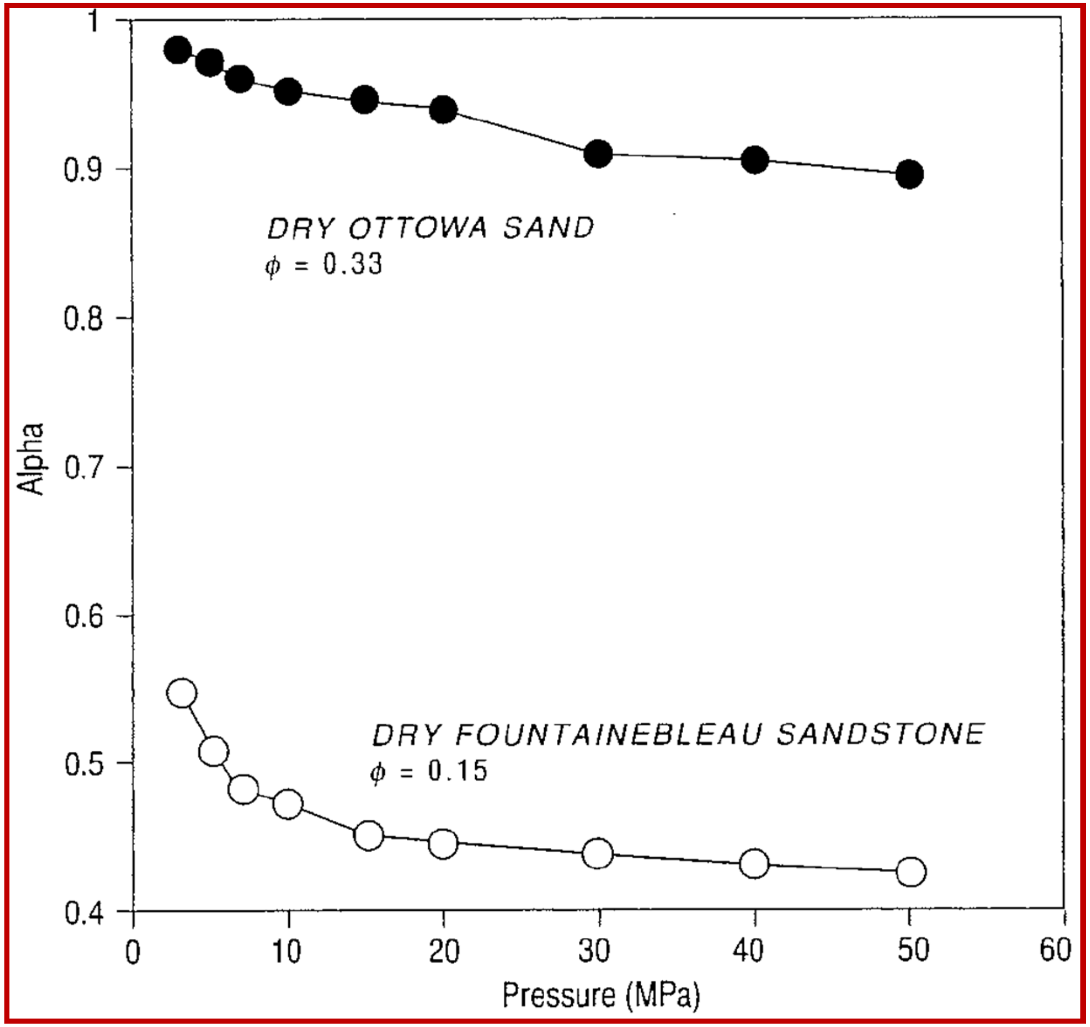
Effective Stress Principle

UFPE

Biot's Effective Stresses

$$\sigma_{ij} = \sigma'_{ij} - \alpha p\ \delta_{ij}$$

- Measured values of α (Biot's parameter) for two porous materials:
  - ✓ Uncemented Sand
  - ✓ Sandstone
- In both cases α decrease with confining pressure

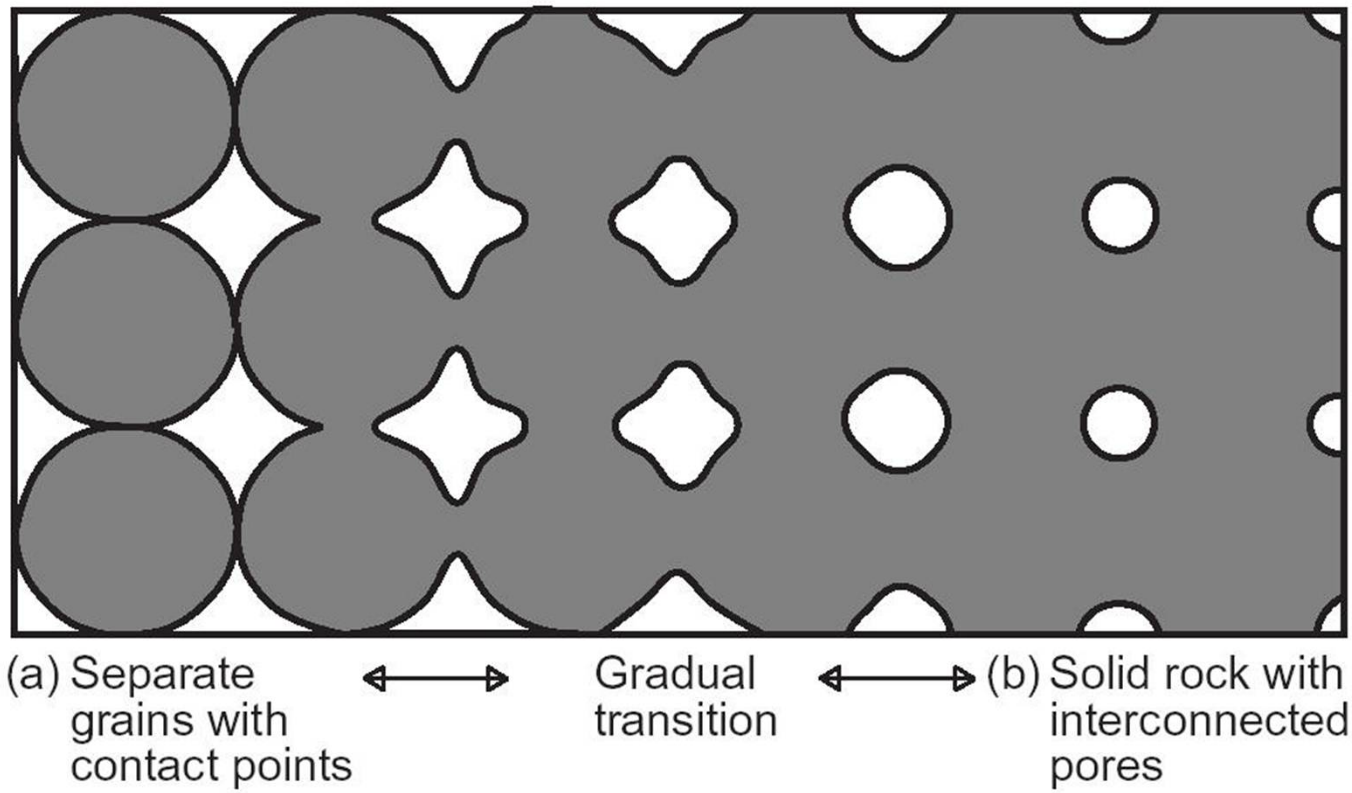


Zoback (2009)

Effective Stress Principle

Biot's Effective Stresses

(Lade & De Boer, 1997)



α □ 1 $0 \leq \alpha \leq 1$ α □ 0

Soil → Rock

Porosity ↘

Strength ↗

Derivation of Biot's Constant
(Lewis and Schrefler, 1998)

Elasticity (solids):

$$d\boldsymbol{\sigma} = \mathbf{D} \cdot d\boldsymbol{\varepsilon}$$

Poro-elasticity for soils: (incompressible grains)

$$d\boldsymbol{\sigma}' = \mathbf{D} \cdot d\boldsymbol{\varepsilon}$$

$$(d\boldsymbol{\sigma}' = d\boldsymbol{\sigma} - dp_f \cdot \mathbf{I})$$

Incremental form

For rocks, we have to consider that the pore pressure p_f induces hydrostatic stress distribution in the solid phase (compressible). The ensuing deformation is a purely volumetric strain:

$$d\varepsilon_v^s = \frac{dp_f}{K_s}$$

or in tensorial form

$$d\boldsymbol{\varepsilon}_v^s = \mathbf{I}\frac{dp_f}{3K_s}$$

The effective stress causes all relevant deformations of the solid skeleton. The constitutive relationship should be rewritten as

Elastic strain tensor ↗ **Total strain tensor** ↗

$$d\boldsymbol{\sigma}' = \mathbf{D} \cdot d\boldsymbol{\varepsilon}^e = \mathbf{D} \cdot (d\boldsymbol{\varepsilon} - d\boldsymbol{\varepsilon}_v^s - d\boldsymbol{\varepsilon}_c - d\boldsymbol{\varepsilon}_T - d\boldsymbol{\varepsilon}^{\text{other mechanisms}}_{\dots}) = \mathbf{D} \cdot (d\boldsymbol{\varepsilon} - d\boldsymbol{\varepsilon}_v^s)$$

(The terms $d\boldsymbol{\varepsilon}_c$, $d\boldsymbol{\varepsilon}_T$ and $d\boldsymbol{\varepsilon}^{\text{other mechanisms}}$ are struck through.)

Derivation of Biot's Constant

(Lewis and Schrefler, 1998)

So, considering the deformations of the solid skeleton as new deformational mechanism:

$$d\boldsymbol{\sigma}' = \mathbf{D}\cdot(d\boldsymbol{\varepsilon} - d\boldsymbol{\varepsilon}_v^s) = \mathbf{D}\cdot d\boldsymbol{\varepsilon} - \mathbf{D}\cdot\mathbf{I}\cdot\frac{dp_f}{3K_s}$$

Biot's definition of effective stress

On the other hand, using Terzaghi's definition of effective stress:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}' + \mathbf{I}\cdot p_f \quad \Rightarrow \quad d\boldsymbol{\sigma} = d\boldsymbol{\sigma}' + \mathbf{I}\cdot dp_f$$

$$\Rightarrow \quad d\boldsymbol{\sigma} = \mathbf{D}\cdot(d\boldsymbol{\varepsilon} - d\boldsymbol{\varepsilon}_v^s) = \mathbf{D}\cdot d\boldsymbol{\varepsilon} - \mathbf{D}\cdot\mathbf{I}\cdot\frac{dp_f}{3K_s} + \mathbf{I}\cdot dp_f$$

Biot's constant: α

$$\Rightarrow \quad d\boldsymbol{\sigma} = \mathbf{D}\cdot d\boldsymbol{\varepsilon} + \left(\mathbf{I} - \mathbf{D}\cdot\mathbf{I}\cdot\frac{1}{3K_s}\right)dp_f$$

$$\Rightarrow \quad d\boldsymbol{\sigma} - \left(1 - \frac{K}{K_s}\right)\cdot\mathbf{I}\cdot dp_f = \mathbf{D}\cdot d\boldsymbol{\varepsilon}$$

Corrected effective stress!!

Derivation of Biot's Constant

(Lewis and Schrefler, 1998)

Biot´s effective stress: $d\boldsymbol{\sigma}''$

$$d\boldsymbol{\sigma} - \left(1 - \frac{K}{K_s}\right) \cdot \mathbf{I} \cdot dp_f = \mathbf{D} \cdot d\boldsymbol{\varepsilon} \quad \Rightarrow \quad d\boldsymbol{\sigma} - \alpha \cdot \mathbf{I} \cdot dp_f = \mathbf{D} \cdot d\boldsymbol{\varepsilon}$$

Biot's constant:

$$\alpha$$

$$d\boldsymbol{\sigma}'' = d\boldsymbol{\sigma} - \mathbf{I} \cdot \alpha \cdot dp_f$$

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}'' + \mathbf{I} \cdot \alpha \cdot p_f$$

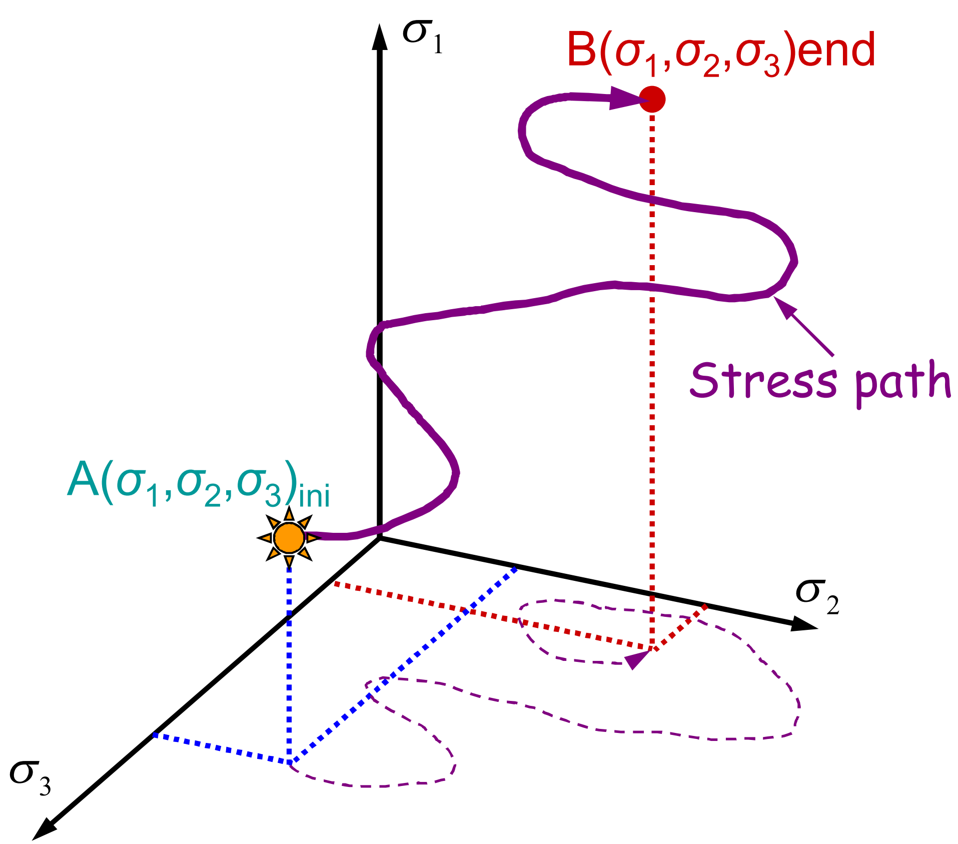
$$\boldsymbol{\sigma}'' = \boldsymbol{\sigma} - \mathbf{I} \cdot \alpha \cdot p_f$$

This is the stress which directly induces rock deformation:

$$d\boldsymbol{\sigma}'' = \mathbf{D} \cdot d\boldsymbol{\varepsilon}$$

Stress Path

❑ Stress path in 3D

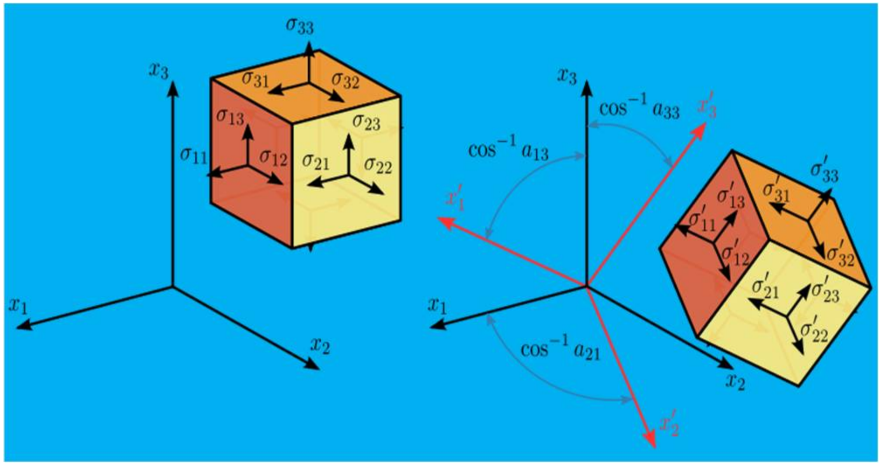


- ✓ It is generally complicate to draw the stress path in 3D
- ✓ We tend to work with invariant if stresses rather with the full stress tensor
- ✓ We use a lot trixial conditions, in which σ_2=σ_3
- ✓ Under this condition we can work in 2D, with σ_1 & σ_3 only
- ✓ This is what we generally do with the Mohr circle.

Stress Invariants

✓ The components of the stress tensor depend on the orientation of the coordinate system at the point under consideration.

✓However, the stress tensor itself is a physical quantity and as such, it is independent of the coordinate system chosen to represent it.



✓There are certain invariants associated with every tensor which are also independent of the coordinate system.

✓ A vector is a simple tensor of rank one. The value of the components will depend on the coordinate system chosen to represent the vector, but the length of the vector is a physical quantity (a scalar) and is independent of the coordinate system chosen to represent the vector.

✓Similarly, every second rank tensor (such as the stress and the strain tensors) has three independent invariant quantities associated with it.

✓One set of such invariants are the principal stresses of the stress tensor.

Stress Invariants

Invariants of the Stress Tensor (σ)

$$I_1 = \sigma_{kk} = \sigma_x + \sigma_y + \sigma_z = \sigma_1 + \sigma_2 + \sigma_3$$

$$I_2 = \frac{1}{2}(\sigma_{ij}\sigma_{ij} - \sigma_{kk}^2) = -(\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z) + \tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2$$

$$= -(\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_3)$$

$$I_3 = \det \boldsymbol{\sigma} = \sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{xz}\tau_{yz} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{xz}^2 - \sigma_z\tau_{xy}^2 = \sigma_1\sigma_2\sigma_3$$

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

Stress Invariants

❑ Stress Deviator Tensor

✓ **The stress tensor can be expressed as the sum of two other stress tensors:**

- **A mean hydrostatic stress tensor or volumetric stress tensor or mean normal stress tensor, which tends to change the volume of the stressed body;**

$$\sigma_m = \frac{1}{3} I_1 = \frac{1}{3}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

- **A deviatoric component called the stress deviator tensor, S, which tends to distort it.**

$$\boldsymbol{\sigma} = \sigma_m \mathbf{I} + \mathbf{s}$$

σ_m → Mean normal stress

$$\boldsymbol{\sigma}_{hydrostatic} = \begin{bmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{bmatrix}$$

$$\boldsymbol{\sigma}_{deviator} = \mathbf{s} = \begin{bmatrix} \sigma_x - \sigma_m & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y - \sigma_m & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z - \sigma_m \end{bmatrix}$$

Stress Invariants

UFPE

Invariants of the stress deviator tensor

$$\mathbf{s} = \boldsymbol{\sigma} - \sigma_m \mathbf{I}$$

$$J_1 = 0$$

= 0 because , the stress deviator tensor is in a state of pure shear

$$J_2 = \frac{1}{2} s_{ij} s_{ij} = -(s_x s_y + s_x s_z + s_y s_z) + \tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2$$

$$= \frac{1}{2}(s_x^2 + s_y^2 + s_z^2) + \tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2$$

$$= \frac{1}{6}\left[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2\right] + \tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2$$

$$J_3 = \det \mathbf{s}$$

$$J_2 = I_2 - \frac{1}{3}(I_1)^2$$

Stress Invariants

Some stress invariants:

Effective mean stress (volumetric behavior):

$$p' = \sigma_m = \bar{\sigma} = \frac{\sigma'_x + \sigma'_y + \sigma'_z}{3}$$

Deviatoric tensor (shear behavior):

$$\mathbf{S} = \begin{pmatrix} s_x & s_{xy} & s_{xz} \\ s_{xy} & s_y & s_{yz} \\ s_{xz} & s_{yz} & s_z \end{pmatrix} = \begin{pmatrix} \sigma_x - \bar{\sigma} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y - \bar{\sigma} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z - \bar{\sigma} \end{pmatrix}$$

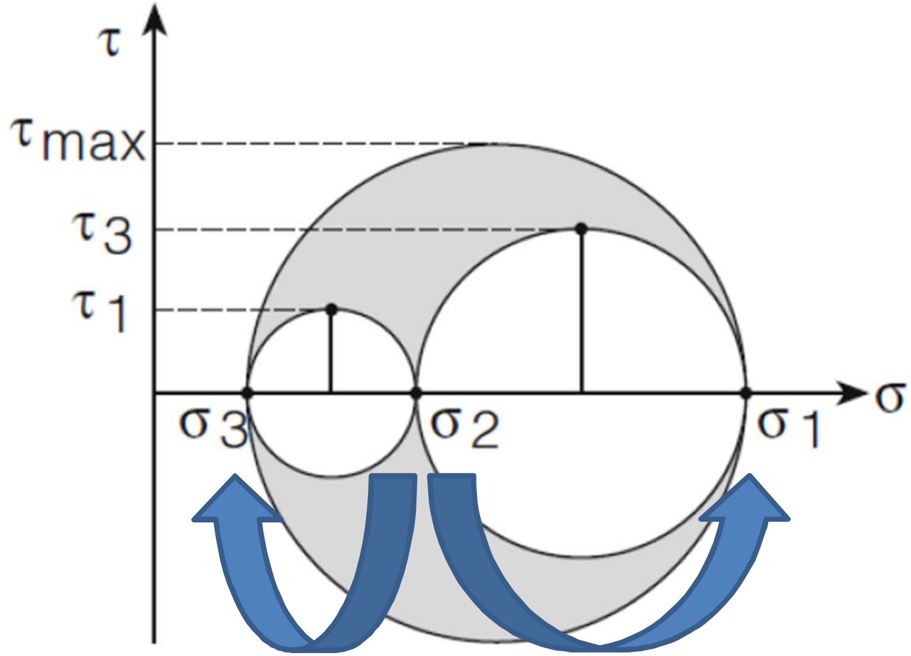
Deviatoric (shear) stress:

$$J = \frac{\sqrt{2}}{2}\sqrt{(\sigma'_x - p')^2 + (\sigma'_y - p')^2 + (\sigma'_z - p')^2 + \tau'^{2}_{xy} + \tau'^{2}_{yz} + \tau'^{2}_{zx}}$$

Lode angle:

$$\theta = -\frac{1}{3}\arcsin\left(\frac{\frac{3\sqrt{3}}{2}\det(\mathbf{S})}{J^3}\right)$$

$$-30^\circ \le \theta \le 30^\circ$$



$\theta = 30°$ | $\theta = -30°$

$\sigma_2 = \sigma_3$ | $\sigma_2 = \sigma_1$

triaxial compression | **triaxial extension**

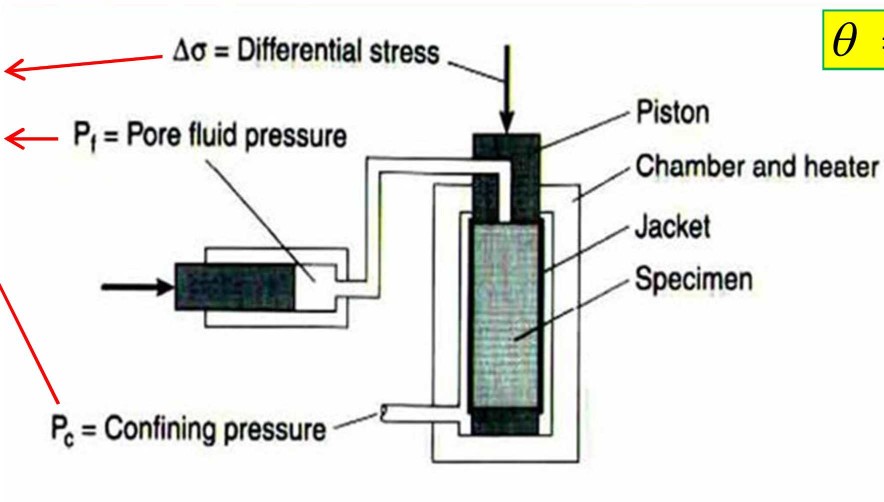
Lode angle:

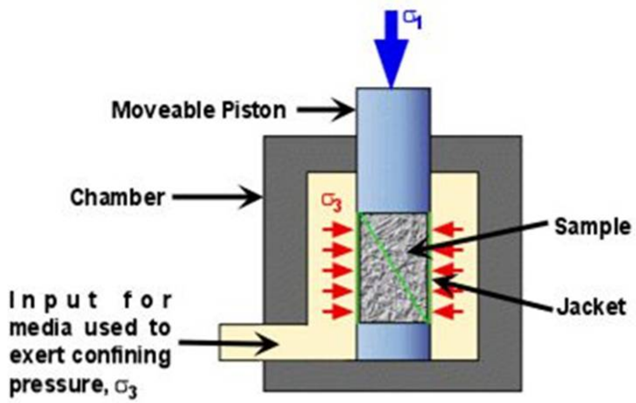
$$\theta = -\frac{1}{3}\arcsin\left(\frac{\frac{3\sqrt{3}}{2}\det(\mathbf{S})}{J^3}\right)$$

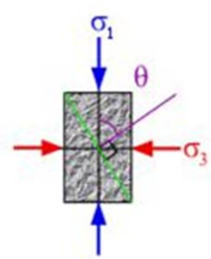
$$-30° \leq \theta \leq 30°$$

Schematic diagram of a triaxial compression apparatus

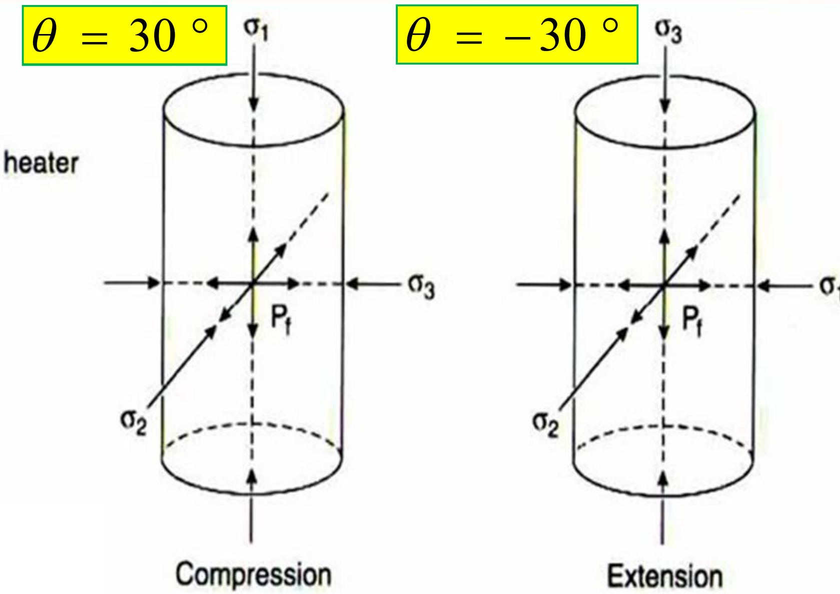
can be varied during the experiment







Stress state in cylindrical specimens in compression and extension tests



Compression:

$\sigma_1^* > \sigma_2^* = \sigma_3^* = P_c - P_f$

$\sigma_1^* = \Delta\sigma + P_c - P_f$

Extension:

$\sigma_3^* < \sigma_1^* = \sigma_2^* = P_c - P_f$

$\sigma_3^* = P_c - P_f - \Delta\sigma$

$\sigma_1^*, \sigma_2^*, \sigma_3^*$ = Maximum, intermediate, minimum effective principal stresses

$P_c - P_f$ = Effective pressure

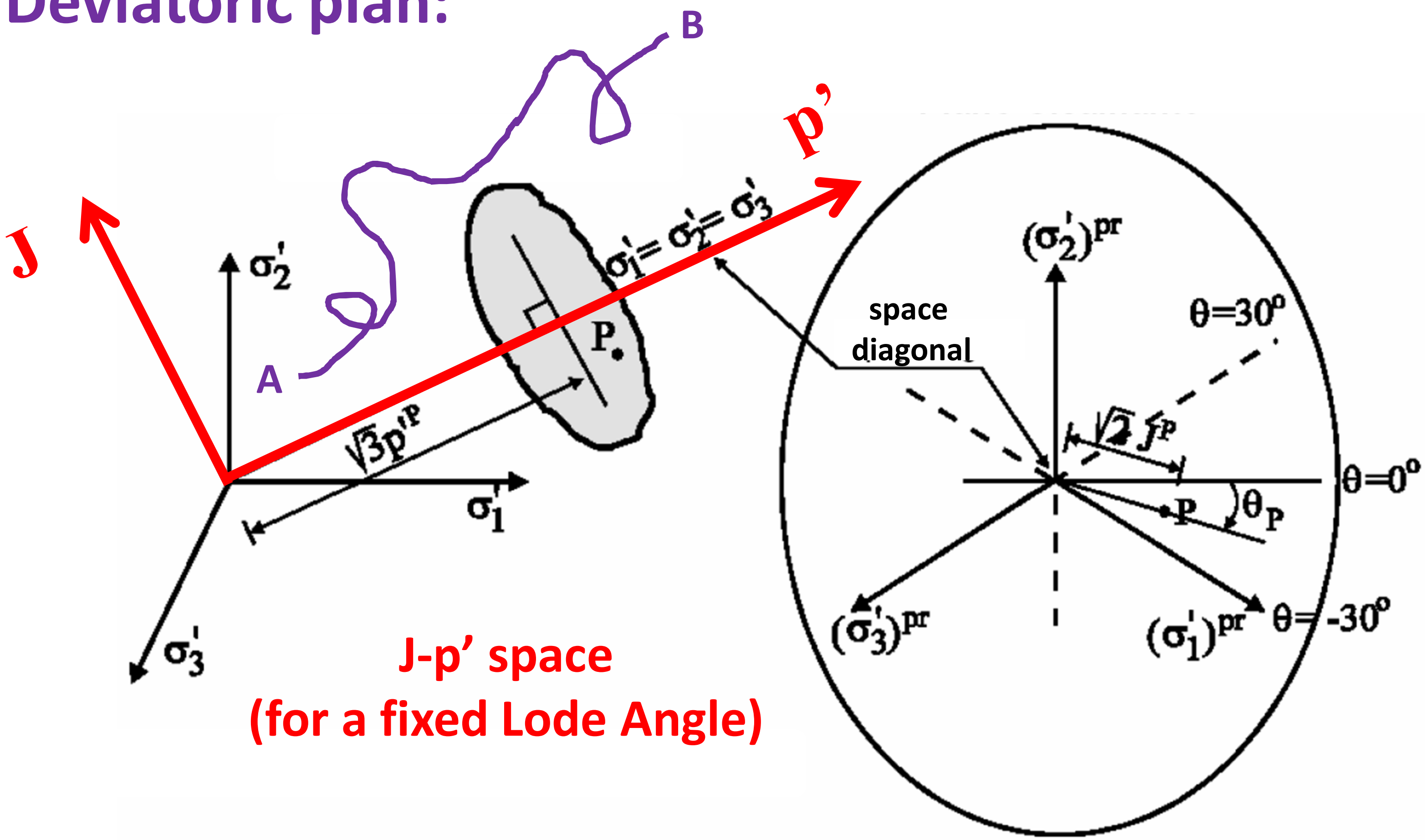
Lode angle:

$$\theta = -\frac{1}{3}\arcsin\left(\frac{\frac{3\sqrt{3}}{2}\det(\mathbf{S})}{J^3}\right)$$

$$-30° \leq \theta \leq 30°$$

Stress Invariants

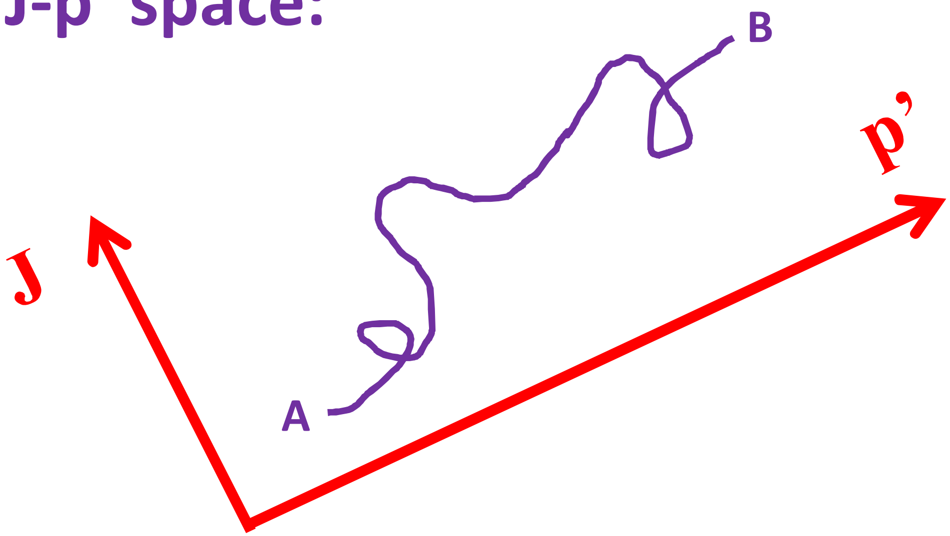
Deviatoric plan:



Potts & Zdravkovic (2001)

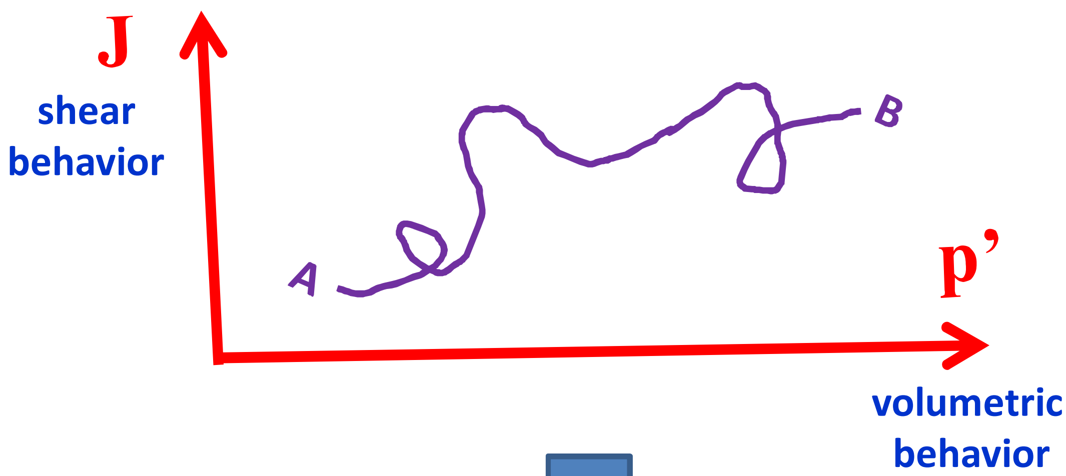
Stress Invariants

J-p’ space:



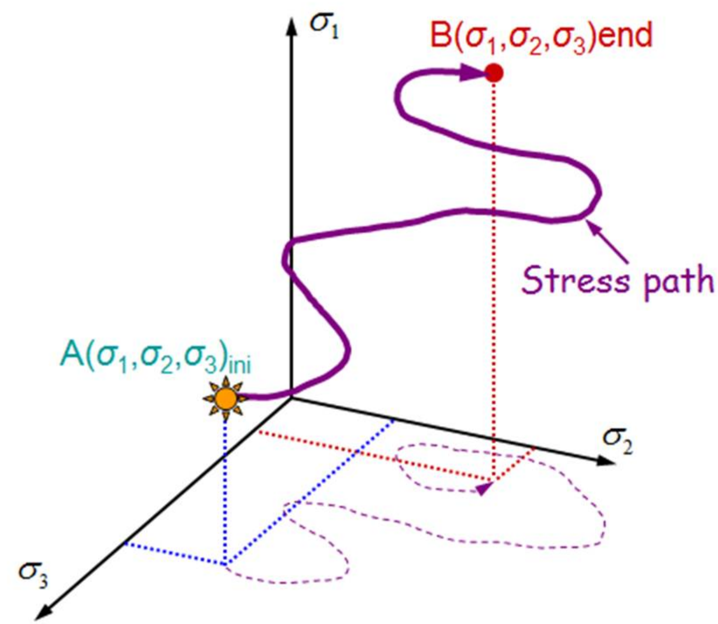
Stress Invariants

J-p’ space:



Better than this...

Material understands J-p’ space but not Cartesian space!

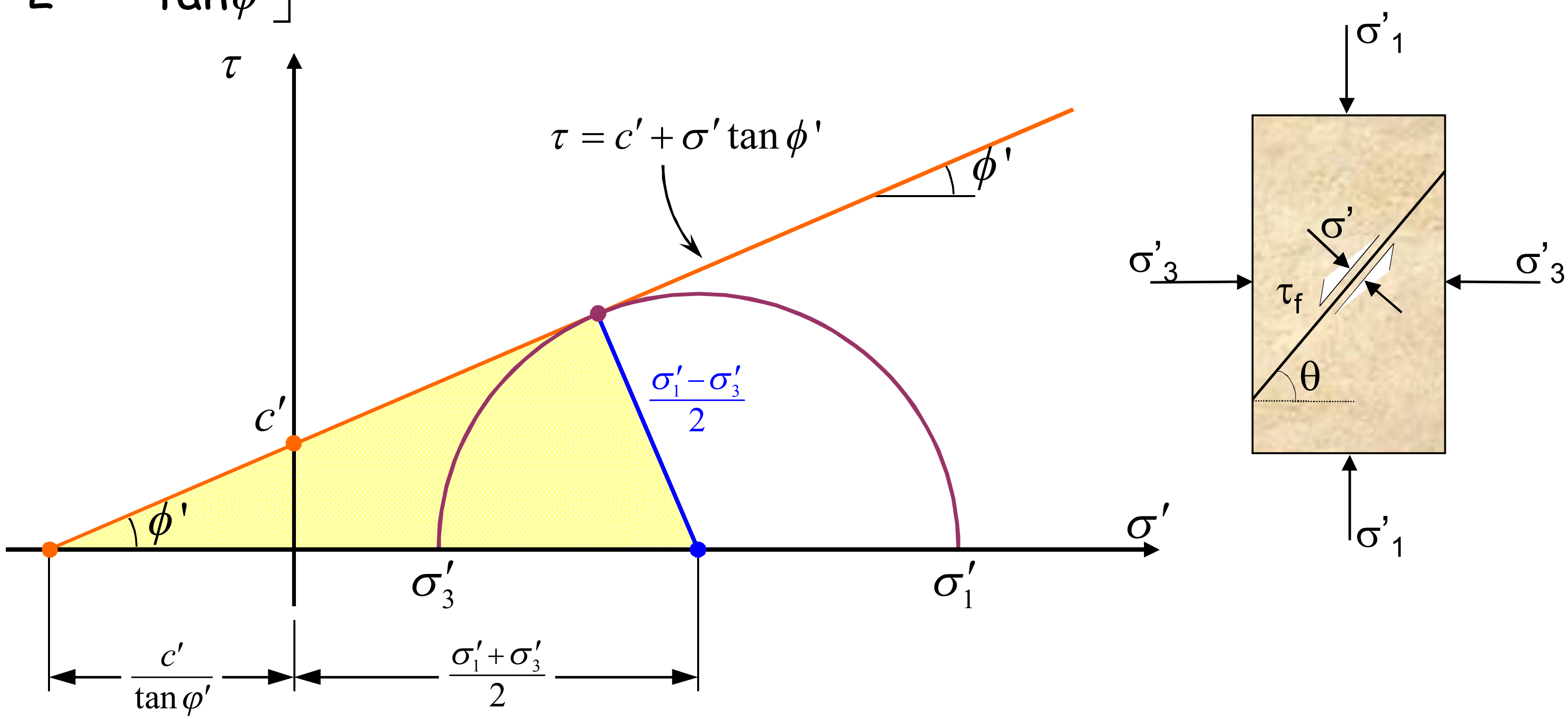


Representation of Constitutive Models

❑ Mohr Coulomb Model in terms of Principal Stresses (2D)

The relationship between the principal stresses at failure and the shear strength parameters is:

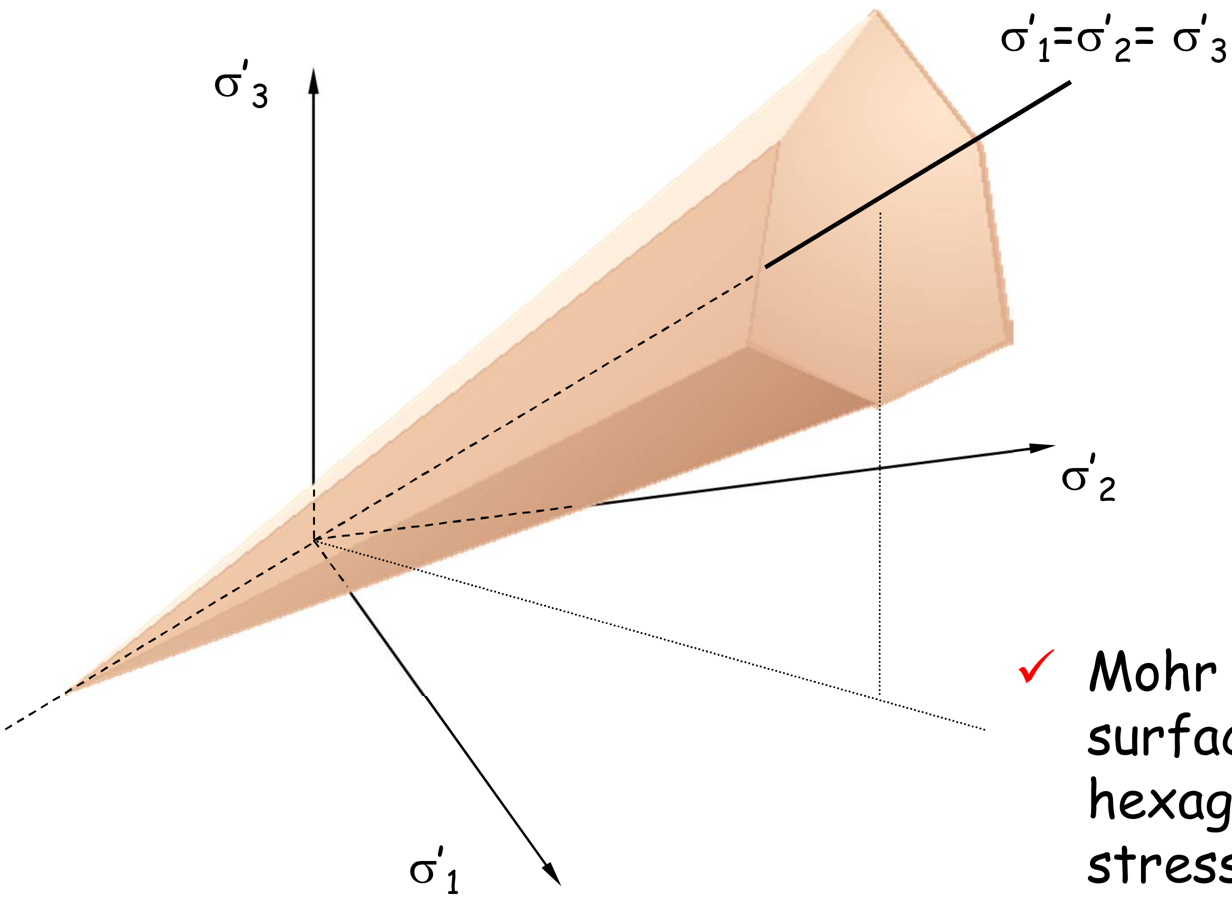
$$\frac{\sigma_1' - \sigma_3'}{2} = \left[\frac{\sigma_1' + \sigma_3'}{2} + \frac{c'}{\tan\phi'}\right]\sin\phi' \qquad (\sigma'_1 - \sigma'_3) = (\sigma'_1 + \sigma'_3)\sin\phi' + 2c'\cos\phi'$$



$$F = (\sigma_1' - \sigma_3') - (\sigma_1' + \sigma_3')\sin\phi' - 2c'\cos\phi' = 0$$

Representation of Constitutive Models

✓ **Mohr Coulomb Model in Principal Stress Space** (3D)

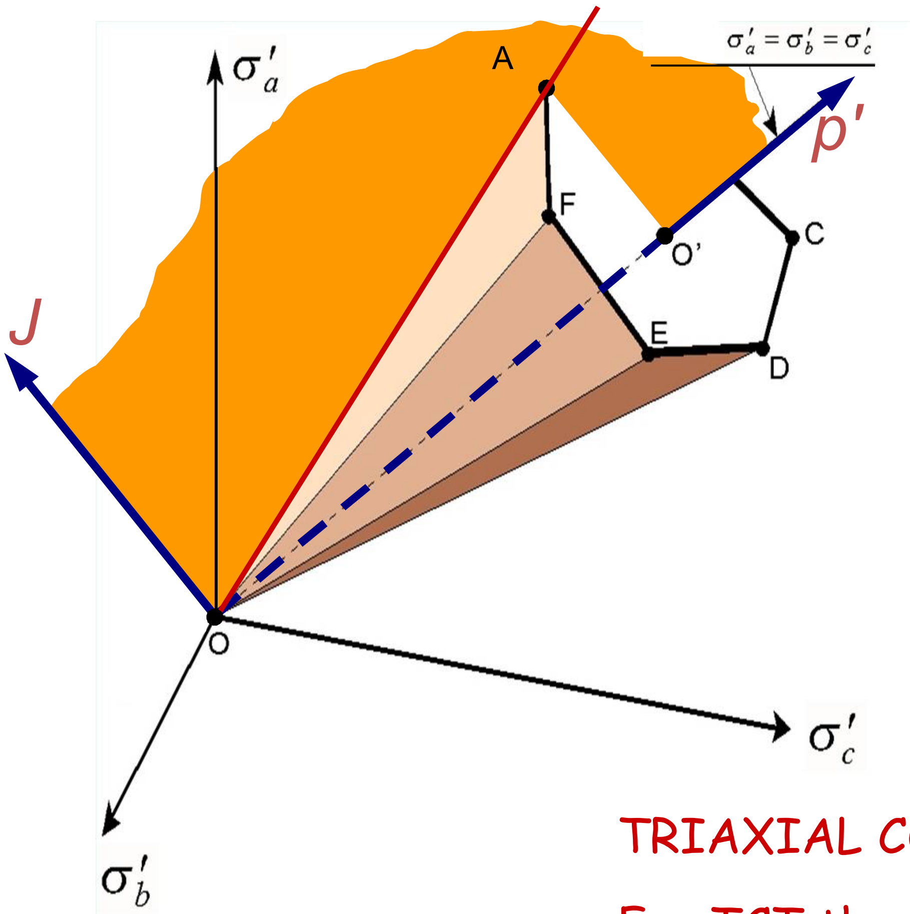


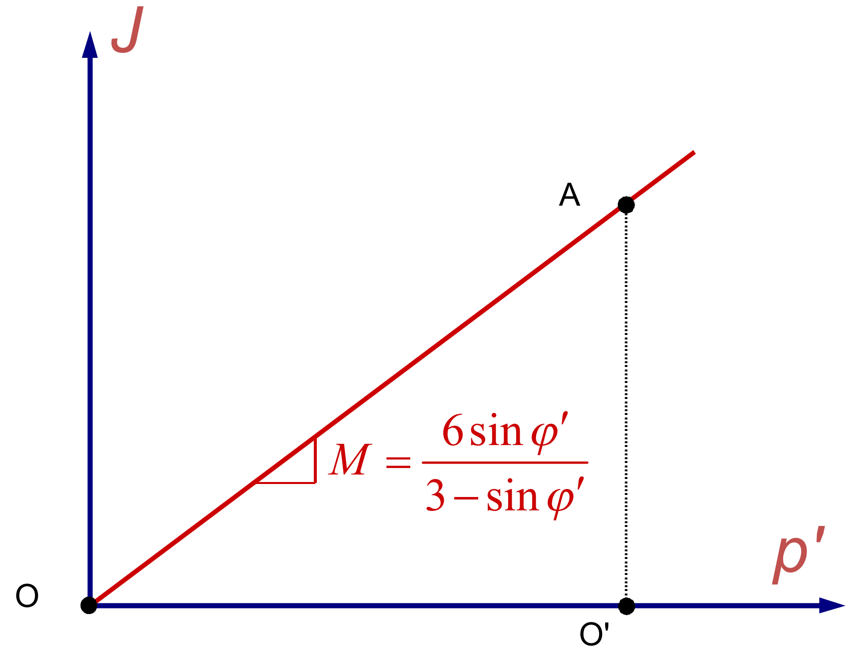
✓ Mohr – Coulomb failure surface is a irregular hexagon in the principal stress space

$$F = (\sigma_1' - \sigma_3') - (\sigma_1' + \sigma_3')\sin\phi' - 2c'\cos\phi' = 0$$

Representation of Constitutive Models

➢ Mohr Coulomb Model in Principal Stress Space (3D)





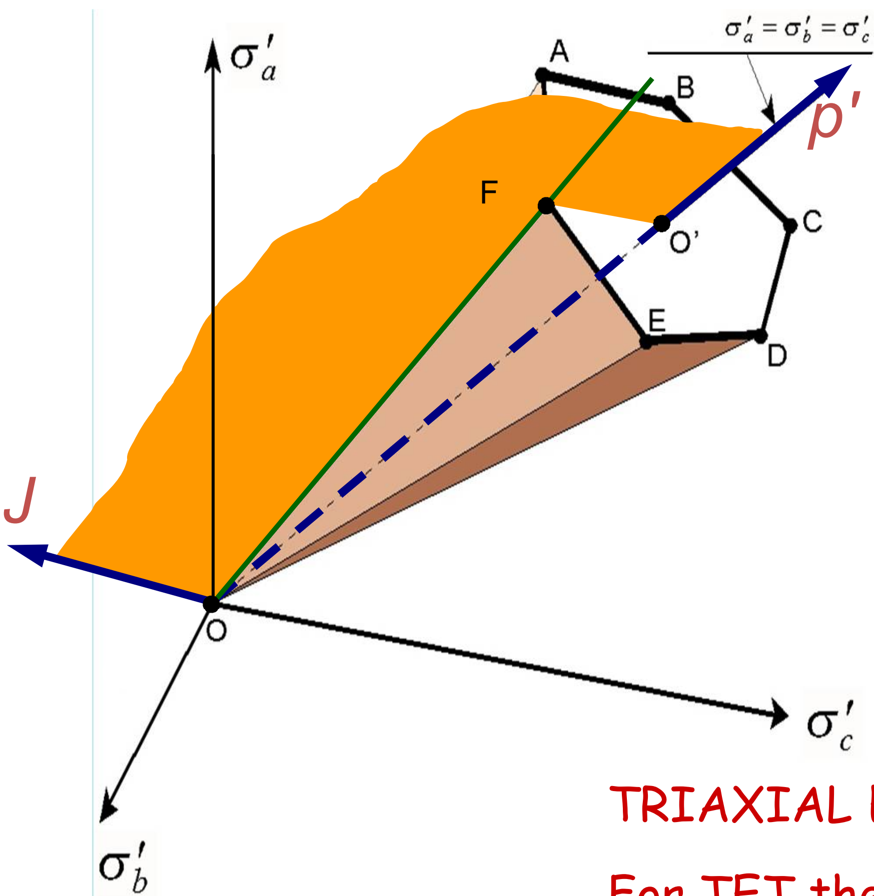
TRIAXIAL COMPRESION TEST (TCT)

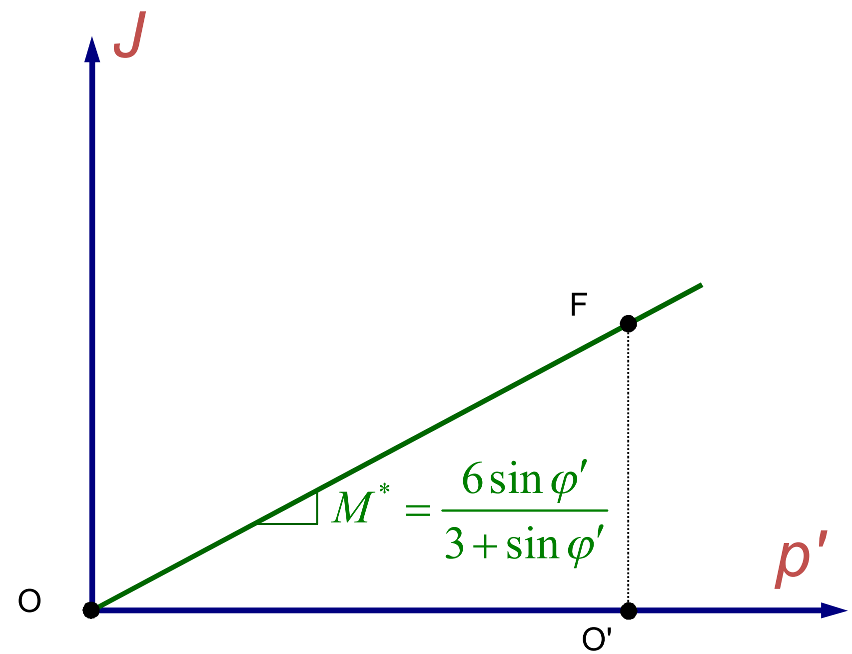
For TCT the Lode Angle: $\Theta = +30°$

The plane (p', J) is the one that pass thought $OO'A$

Representation of Constitutive Models

➤ **Mohr Coulomb Model in Principal Stress Space (3D)**





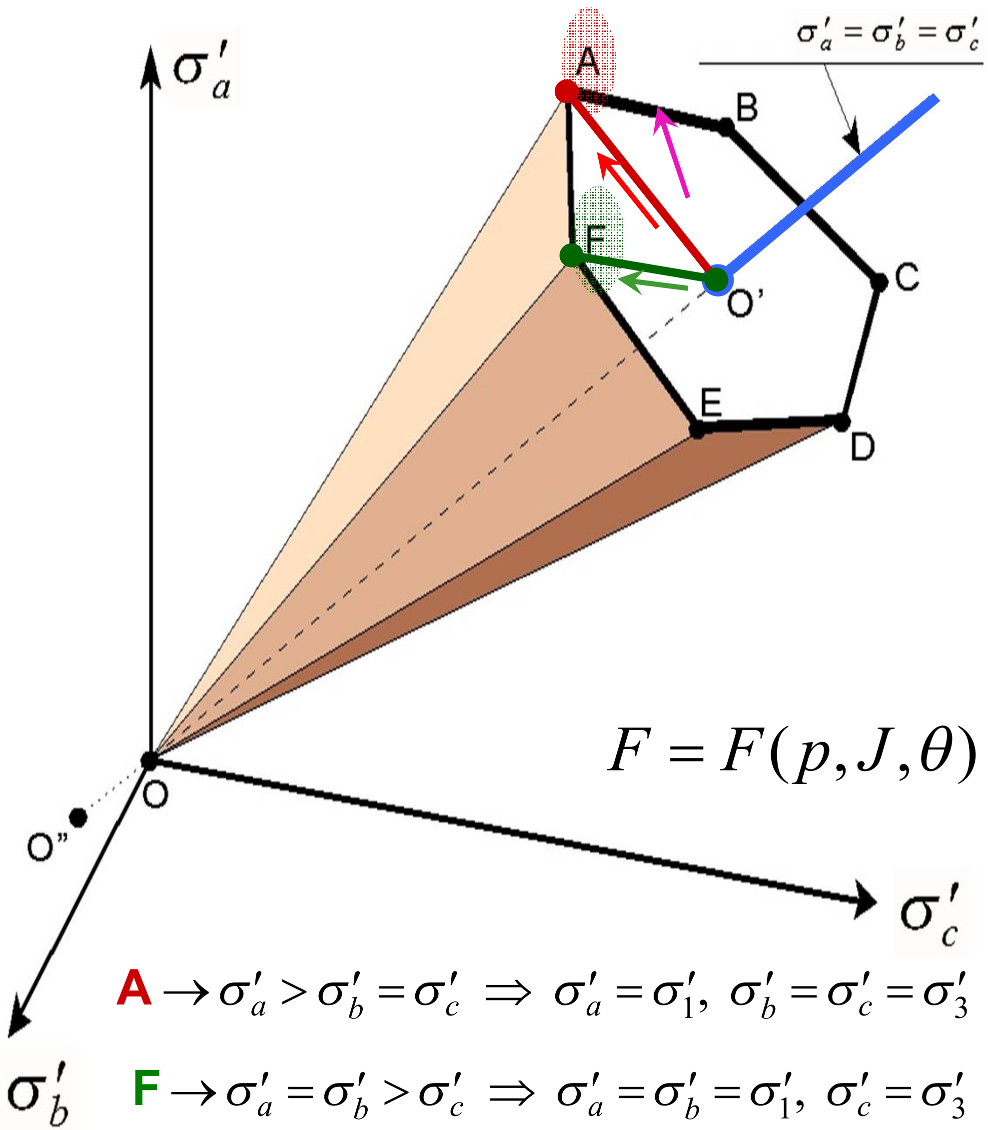
TRIAXIAL EXTENSION TEST (TET)

For TET the Lode Angle: Θ = - 30°

The plane (p',q) is the one that pass thought OO'F

UFPE

➢ Mohr Coulomb Model in Principal Stress Space (3D)



$$F = F(p, J, \theta)$$

A $\rightarrow \sigma'_a > \sigma'_b = \sigma'_c \Rightarrow \sigma'_a = \sigma'_1,\ \sigma'_b = \sigma'_c = \sigma'_3$

F $\rightarrow \sigma'_a = \sigma'_b > \sigma'_c \Rightarrow \sigma'_a = \sigma'_b = \sigma'_1,\ \sigma'_c = \sigma'_3$

- ✓ Yielding/failure for TCT (OO'A) corresponds to a stress path with θ=+30°. ↑
- ✓ Yielding/failure for TET (OO'F) corresponds to a stress path with θ=-30°. ↑
- ✓ We may need to predict yielding or failure for any stress path (i.e. any θ) ↑
- ✓ We can use a function g(θ) that generalize the yield/failure surface to any stress path (i.e. any θ)

$$F \equiv J - (-p + a)g(\theta) = 0; \quad where$$

$$g(\theta) = \frac{\sin\phi}{\cos\theta + \frac{1}{\sqrt{3}}\sin\theta\sin\phi}; \quad a = \frac{c}{g(\theta)}$$

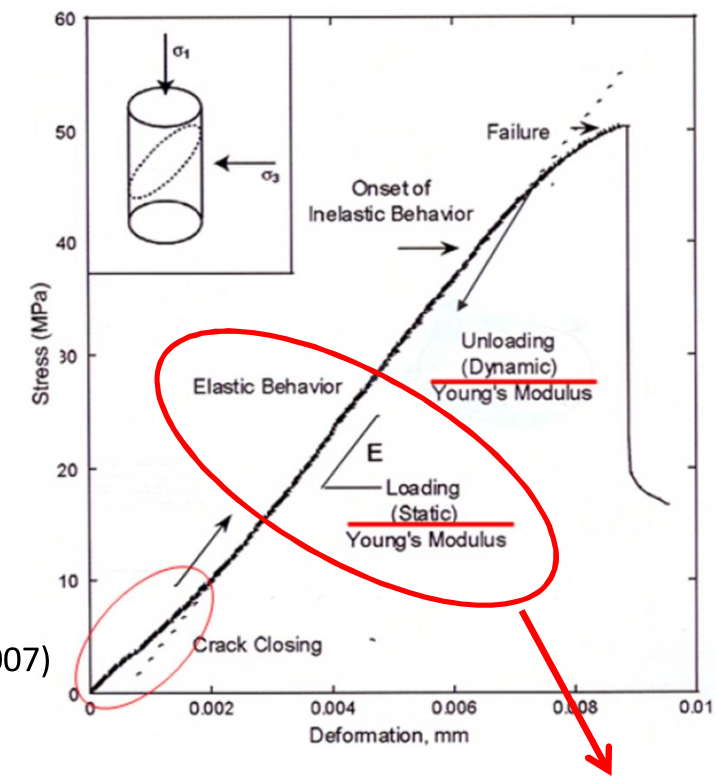
Mechanical Constitutive Behaviour

More about stress-strain relationship...

$$\boldsymbol{\sigma}' = \mathbf{D} \cdot \boldsymbol{\varepsilon}$$

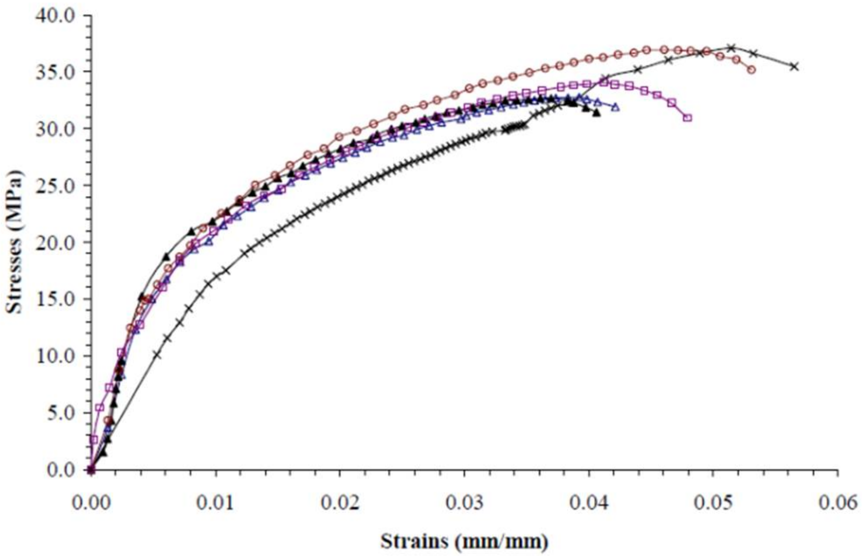
Linear Elasticity – Isotropic Materials

Compression test:



(Zoback, M. D., 2007)

Uniaxial compression tests in reservoir rocks:



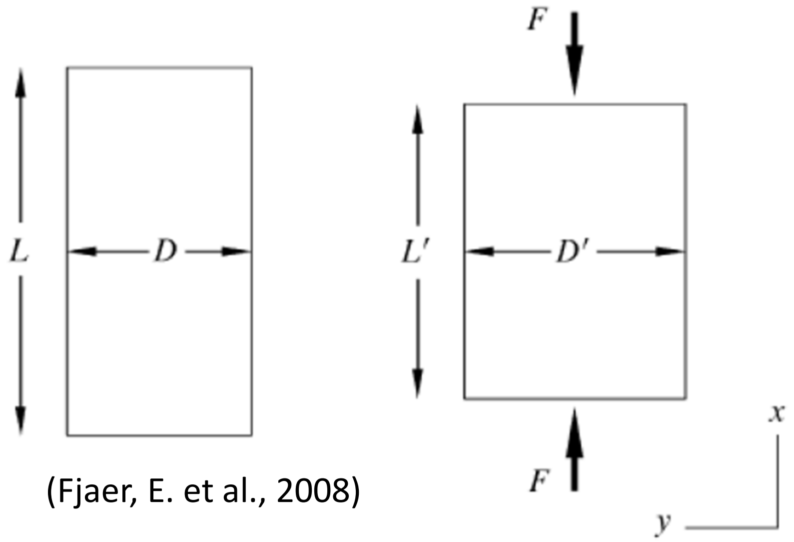
(Jandakaew, M. and Chevrom, 2007)

Young modulus:

$$\varepsilon_x = \frac{1}{E}\sigma_x$$

Linear Elasticity – Isotropic Materials

Deformed sample subjected to uniaxial stress:



(Fjaer, E. et al., 2008)

$$\sigma_y = \sigma_z = \tau_{xy} = \tau_{xz} = \tau_{yz} = 0$$

$$\varepsilon_x = \frac{L - L'}{L} = -\frac{\Delta L}{L} \quad < 0$$

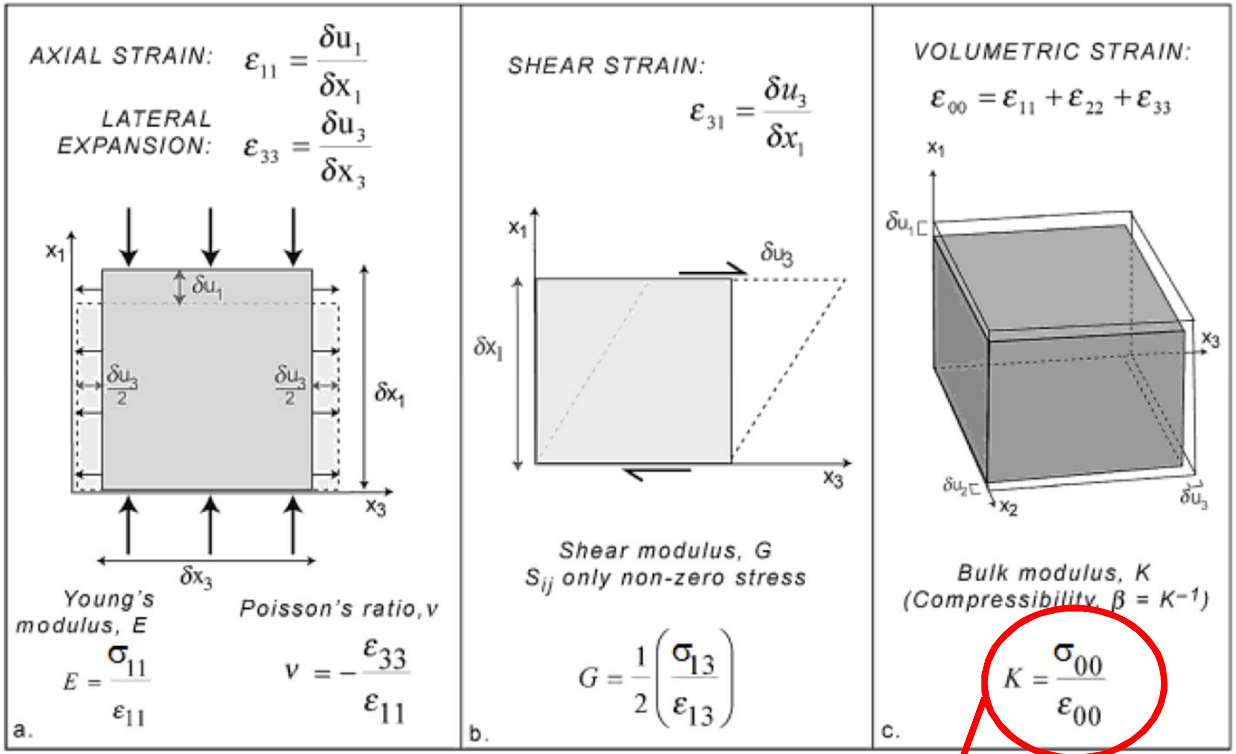
$$\varepsilon_y = \frac{D - D'}{D} \quad > 0$$

Poisson ratio:

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}$$

Linear Elasticity – Isotropic Materials

Physical interpretation of elastic modulus:



(Zoback, M. D., 2007)

$$p = K\varepsilon_v \quad \begin{cases} p = \text{mean stress} \\ K = \text{bulk modulus} \\ \varepsilon_v = \text{volumetri c strain} \end{cases}$$

Linear Elasticity – Isotropic Materials

Relationship between elastic modulus:

$E = 3K(1-2\nu)$	$K = \lambda\frac{1+\nu}{3\nu}$	$\frac{\lambda}{\lambda+G} = 2\nu$
$E = 2G(1+\nu)$	$K = \frac{2}{3}G\frac{1+\nu}{1-2\nu}$	$\frac{G}{\lambda+G} = 1-2\nu$
$E = \frac{9KG}{3K+G}$	$K = \lambda + \frac{2}{3}G$	$\frac{\lambda+2G}{\lambda+G} = 2(1-\nu)$
$E = G\frac{3\lambda+2G}{\lambda+G}$	$K = \frac{GE}{9G-3E}$	$\frac{3\lambda+2G}{\lambda+G} = 2(1+\nu)$
$E = \frac{\lambda}{\nu}(1+\nu)(1-2\nu)$	$\frac{\lambda}{G} = \frac{2\nu}{1-2\nu}$	$\frac{3\lambda+4G}{\lambda+G} = 2(2-\nu)$
$H = \lambda + 2G$	$H = K + \frac{4}{3}G$	$\nu = \frac{3K-2G}{2(3K+G)}$
$H = E\frac{1-\nu}{(1+\nu)(1-2\nu)}$	$H = 2G\frac{1-\nu}{1-2\nu}$	$H = 3K\frac{1-\nu}{1+\nu}$

Lamé constants:

$$\lambda \qquad G$$

Uniaxial compaction modulus:

$$H$$

Bulk modulus:

$$K = \frac{\sigma_p}{\varepsilon_{vol}} = \lambda + \frac{2}{3}G$$

Linear Elasticity – Isotropic Materials

Homogeneous isotropic material:

$$\sigma_x = (\lambda + 2G)\varepsilon_x + \lambda\varepsilon_y + \lambda\varepsilon_z$$

$$\sigma_y = \lambda\varepsilon_x + (\lambda + 2G)\varepsilon_y + \lambda\varepsilon_z$$

$$\sigma_z = \lambda\varepsilon_x + \lambda\varepsilon_y + (\lambda + 2G)\varepsilon_z$$

$$\tau_{yz} = 2G\mathit{\Gamma}_{yz}$$

$$\tau_{xz} = 2G\mathit{\Gamma}_{xz}$$

$$\tau_{xy} = 2G\mathit{\Gamma}_{xy}$$

or:

$$\sigma_{ij} = \lambda\varepsilon_{vol}\delta_{ij} + 2G\varepsilon_{ij}$$

$$\delta_{ij} \overset{def}{=} \begin{cases} 1 & \text{se } i = j \\ 0 & \text{se } i \neq j \end{cases}$$

Kronecker delta

Using tensorial notation:

$$\boldsymbol{\sigma} = \mathbf{D} \cdot \boldsymbol{\varepsilon}$$ or $$d\boldsymbol{\sigma} = \mathbf{D} \cdot d\boldsymbol{\varepsilon}$$ (incremental form)

where: $\mathbf{D}(E, \nu)$ **is the elastic constitutive tensor that relates the stress and strain tensors.**

Anisotropy

If the elastic response of a material is not independent of the material's orientation for a given stress configuration, the material is said to be anisotropic. Thus the elastic moduli of an anisotropic material are different for different directions in the material.

Most rocks are anisotropic to some extent…

The origin of the anisotropy is always heterogeneities on a smaller scale than the volume under investigation.

Sedimentary rocks are created during a deposition process where the grains normally are not deposited randomly. Seasonal variations in the fluid flow rates may result in alternating microlayers of fine and coarser grain size distributions.

Due to its origin, anisotropy of this type is said to be lithological or intrinsic. Another important type is anisotropy induced by external stresses. The anisotropy is then normally caused by microcracks, generated by a deviatoric stress and predominantly
oriented normal to the lowest principal stress.

Note (Fjaer et al., 2008): In calculations on rock elasticity, anisotropy is often ignored. This simplification may be necessary rather than just comfortable, because—as we shall see—an anisotropic description requires much more information about the material—information that may not be available. However, by ignoring anisotropy, one may in some cases introduce large errors that invalidate the calculations.

Anisotropy

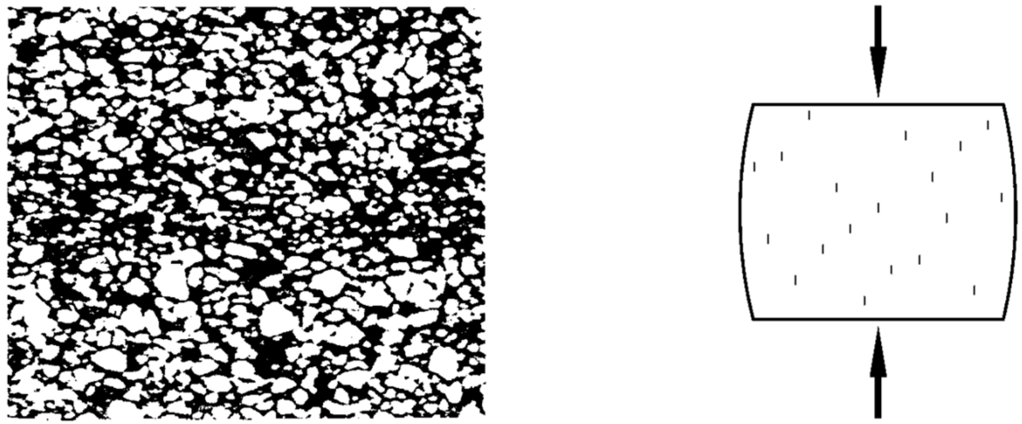


Illustration of intrinsic (lithological) and stress induced anisotropy

For a general anisotropic material, each stress component is linearly related to every strain component by independent coefficients:

$$\sigma_{ij} = \sum_{k,l} C_{ijkl} \varepsilon_{kl}$$

Since the indices i, j, k and l may each take the values 1,2 or 3, there are all together 81 of the constants C_{ijkl}

Some of these vanish and others are equal by symmetry, however, so that the number of independent constants is considerably less: $C_{ijkl} = C_{jikl} = C_{ijlk} = C_{jilk}$ **and** $C_{ijkl} = C_{klij}$

with that, the number of independent constants reduces to 21.

Anisotropy: orthorhombic symmetry

Orthorhombic symmetry: Rocks can normally be described reasonably well by assuming that the material has three mutually perpendicular planes of symmetry.

$$
\begin{aligned}
\sigma_x &= C_{11}\varepsilon_x + C_{12}\varepsilon_y + C_{13}\varepsilon_z & \tau_{yz} &= 2C_{44}\Gamma_{yz} \\
\sigma_y &= C_{12}\varepsilon_x + C_{22}\varepsilon_y + C_{23}\varepsilon_z & \tau_{xz} &= 2C_{55}\Gamma_{xz} \\
\sigma_z &= C_{13}\varepsilon_x + C_{23}\varepsilon_y + C_{33}\varepsilon_z & \tau_{xy} &= 2C_{66}\Gamma_{xy}
\end{aligned}
$$

or using vetorial notation of stress and strains:

$$
\boldsymbol{\sigma} = \mathbf{C} \cdot \boldsymbol{\varepsilon} \qquad \text{where} \quad \mathrm{C} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{pmatrix} \quad \boldsymbol{\sigma} = \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{pmatrix}, \qquad \boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ 2\Gamma_{yz} \\ 2\Gamma_{xz} \\ 2\Gamma_{xy} \end{pmatrix}
$$

These stress–strain relations generally describe most types of rocks.

This model describes the elastic properties of any linear elastic material with or-thorhombic or higher symmetry. Thus they may also describe an isotropic rock:

$$
C_{11} = C_{22} = C_{33} = \lambda + 2G \qquad C_{12} = C_{13} = C_{23} = \lambda \qquad C_{44} = C_{55} = C_{66} = G
$$

Anisotropy: orthorhombic symmetry

Example: consider the uniaxial stress state defining Young's modulus and Poisson's ratio. In this example, $\sigma_y = \sigma_z = 0$ and $\tau_{xy} = \tau_{xz} = \tau_{yz} = 0$. The stress–strain relations become:

$$\sigma_x = C_{11}\varepsilon_x + C_{12}\varepsilon_y + C_{13}\varepsilon_z \quad (1)$$

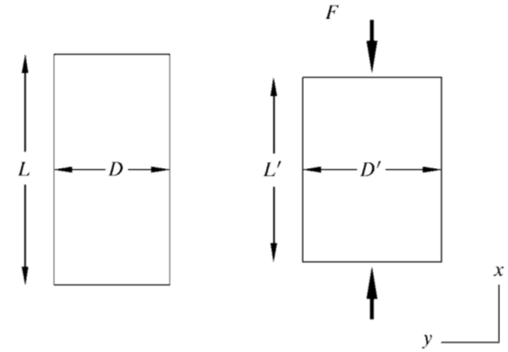
$$0 = C_{12}\varepsilon_x + C_{22}\varepsilon_y + C_{23}\varepsilon_z \quad (2)$$

$$0 = C_{13}\varepsilon_x + C_{23}\varepsilon_y + C_{33}\varepsilon_z \quad (3)$$

$$0 = 2C_{44}\Gamma_{yz} \quad (4)$$

$$0 = 2C_{55}\Gamma_{xz} \quad (5)$$

$$0 = 2C_{66}\Gamma_{xy} \quad (6)$$



Deformation induced by uniaxial stress.

Solving the equations above (2,3) for $\nu = -\varepsilon_y/\varepsilon_x$, we find

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} = \frac{C_{12}C_{33} - C_{13}C_{23}}{C_{22}C_{33} - C_{23}^2}$$

while for $\nu = -\varepsilon_z/\varepsilon_x$ we find (by interchanging indices 2 and 3):

$$\nu = -\frac{\varepsilon_z}{\varepsilon_x} = \frac{C_{13}C_{22} - C_{12}C_{23}}{C_{22}C_{33} - C_{23}^2}$$

Thus the value of Poisson's ratio depends not only on the direction of the applied stress, but also on the direction in which lateral expansion is measured.

Anisotropy: transverse isotropy

UFPE

Transverse isotropy: A special type of symmetry, which is relevant for many types of rocks, is full rotational symmetry around one axis. Rocks possessing such symmetry are said to be Transversely isotropic. It implies that the elastic properties are equal for all directions within a plane, but different in the other directions. This extra element of symmetry reduces the number of independent elastic constants to 5.

Assuming that the x- and y-directions are equivalent while the z-direction is the different one, we may rotate the coordinate system any angle around the z-axis without altering the elastic constants. For this to be possible it is required that $C_{11} = C_{22}$, $C_{13} = C_{23}$, $C_{12} = C_{11} - 2C_{66}$, and $C_{44} = C_{55}$. The stiffness matrix for a transversely isotropic material having the z-axis as the unique axis is then

$$\mathbf{C} = \begin{pmatrix} C_{11} & C_{11}-2C_{66} & C_{13} & 0 & 0 & 0 \\ C_{11}-2C_{66} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{pmatrix}$$

Note: linear elasticity

$$\mathbf{C} = \begin{pmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{pmatrix}$$

2 constants

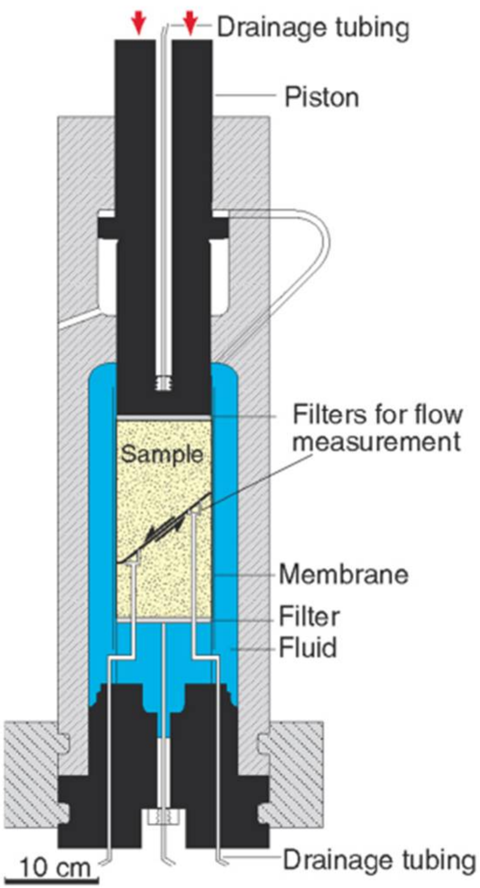
Transverse isotropy is normally considered to be a representative symmetry for horizontally layered sedimentary rocks.

Stress induced anisotropy may often be described by transverse isotropy as well.

Realistic stress-strain relationship

Realistic stress-strain relationships: based on experiments

The experimental apparatus:



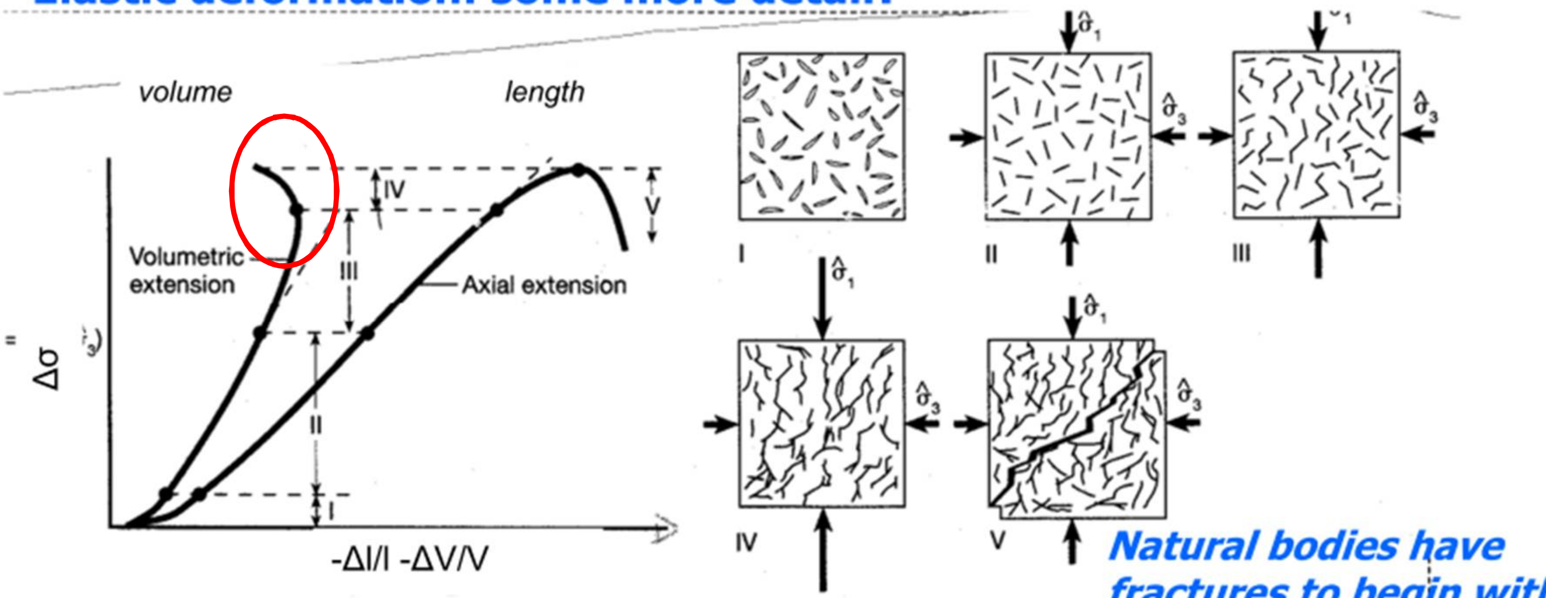
- temperatures typically up to >1000ºC
- high pressures more challenging and results more questionable
- strain normally < 10%
- Typical duration of experiments < weeks
- typical strain rates = 10^5 sec^{-1} (in nature ca. 10^{-15} - 10^{-14})

Experiments are under *uniaxial* or *triaxial* (rare)

Stress is applied along the axis of the sample with varying confining pressure (Pc).

Before Failure

Elastic deformation: some more detail?



Natural bodies have fractures to begin with

Stage I: **the slope of the curve is low and tends to increase (more Δσ needed per unit deformation). The volume decreases *(fractures close)***

Stages II-III: **The sample follows the correct elastic behaviour** ***Fractures are closed and new fractures are created spread in the body.***

Stage IV: **Things get "out-of-hand" and the moment of failure is approaching. More and more fractures are created and start linking creating longer fractures. Volume starts increasing (dilatancy)**

Very important!! (associated to shear FAILURE)

Rock Failure

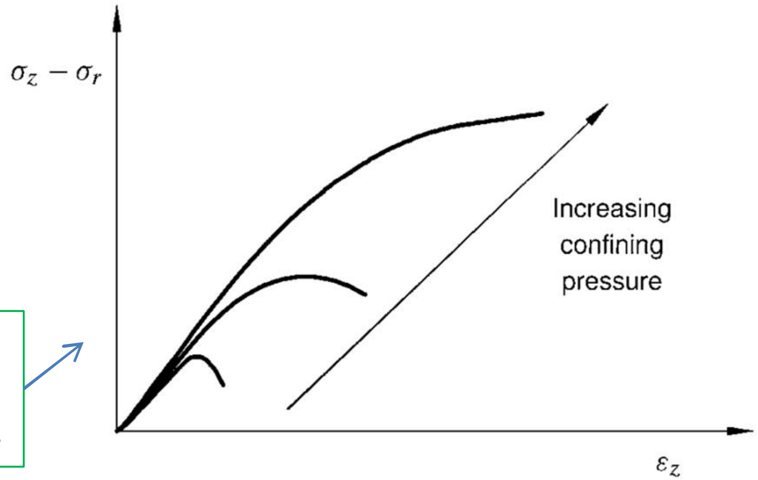
The strength of rocks
=
The stress you need to apply to have failure

Various factors

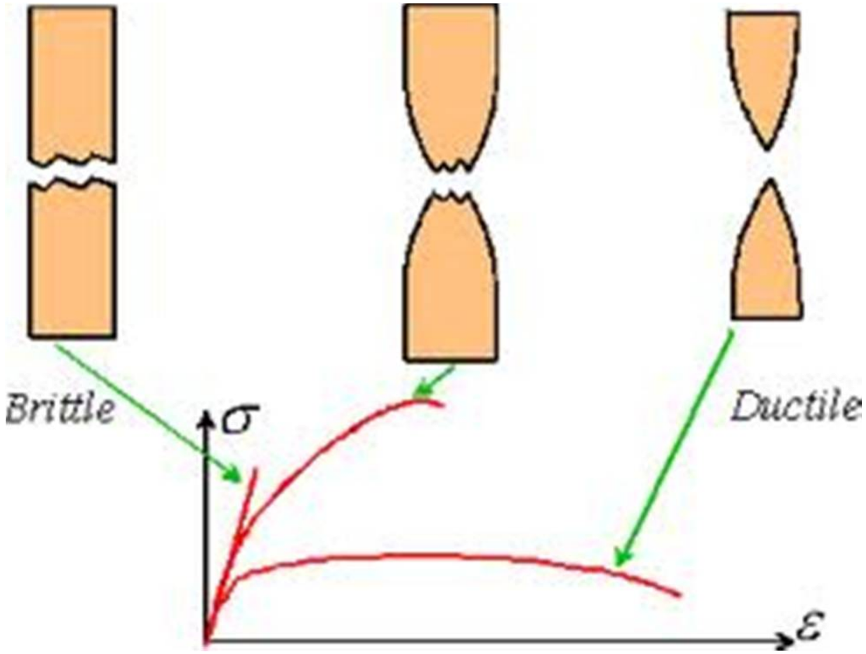
- **Lithology: Important but typically overshadowed by other factors such as porosity, state of alteration etc etc**
- **Temperature: Not important as long as deformation remains in the brittle field**
- **strain rate: not very important**
- **Anisotropy of rocks: Quite important!**

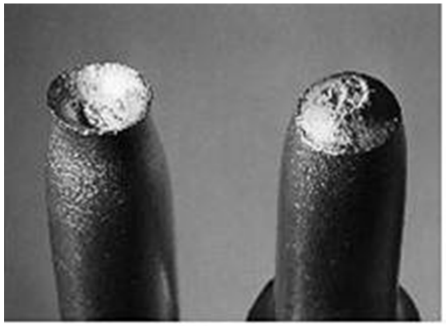
- **confining pressure: THE MOST IMPORTANT**

Triaxial testing: typical influence of the confining pressure on the shape of the differential stress (axial stress minus confining pressure) versus axial strain curves



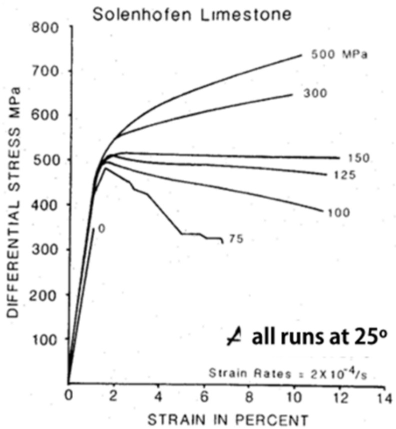
Realistic stress-strain relationship

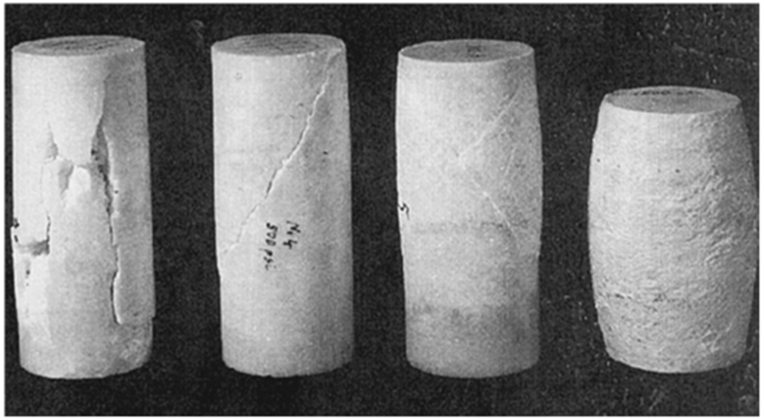




Rocks:

It depends on the confining stress!!

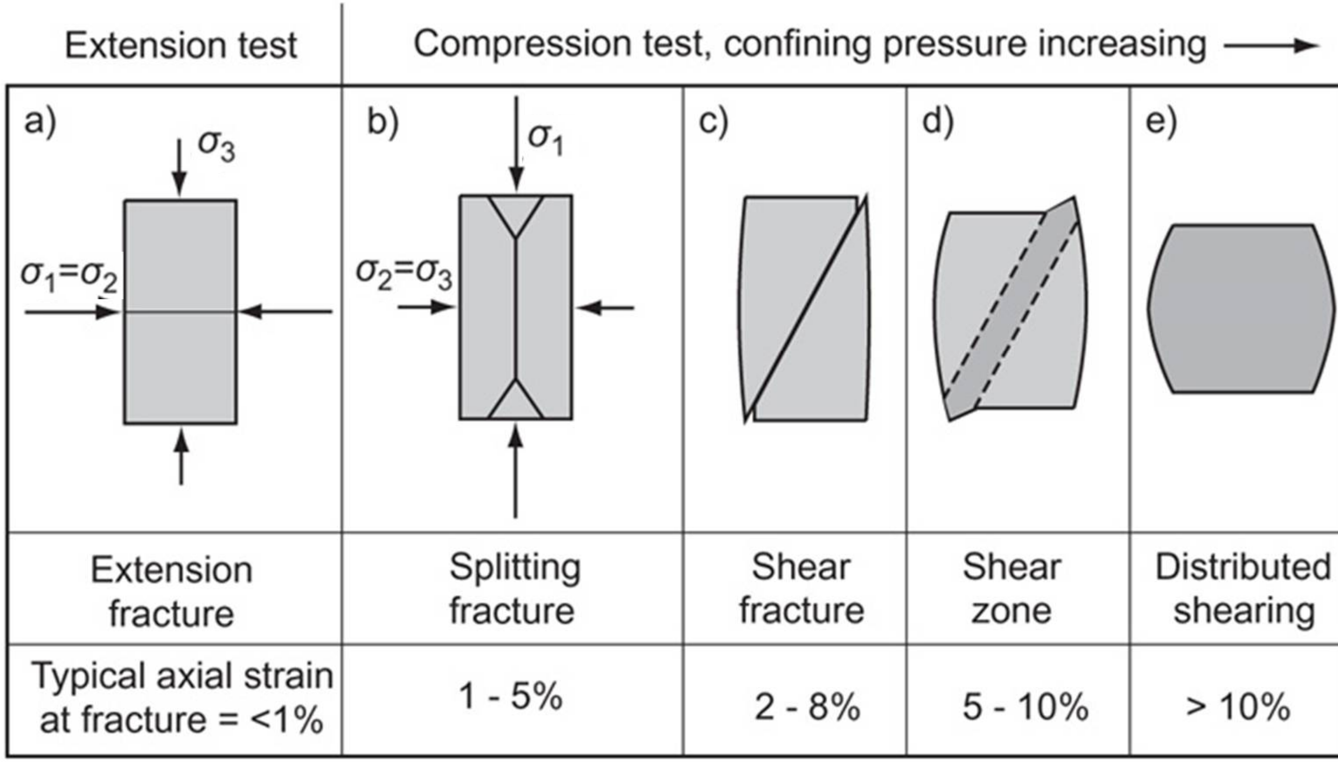




Schematic representation of brittle failure styles in triaxial tests (Griggs and Handin, 1960a).

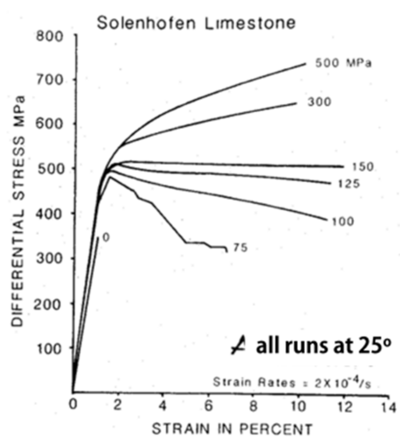
a) Extension test.

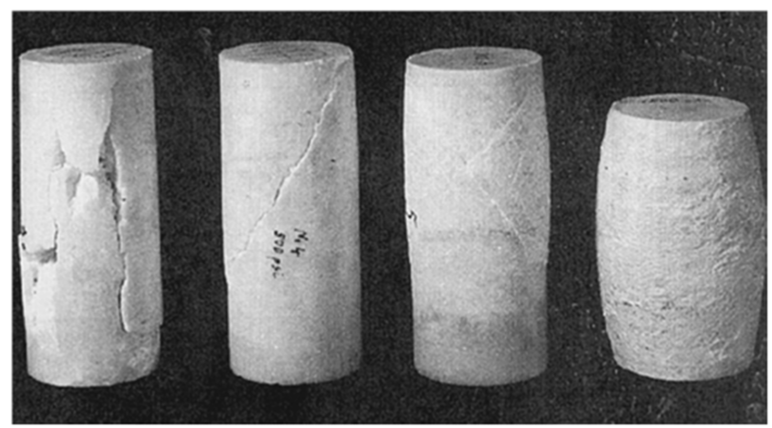
b) – e) Compression test with confining pressure increasing to the right.



Rocks:

It depends on the confining stress!!

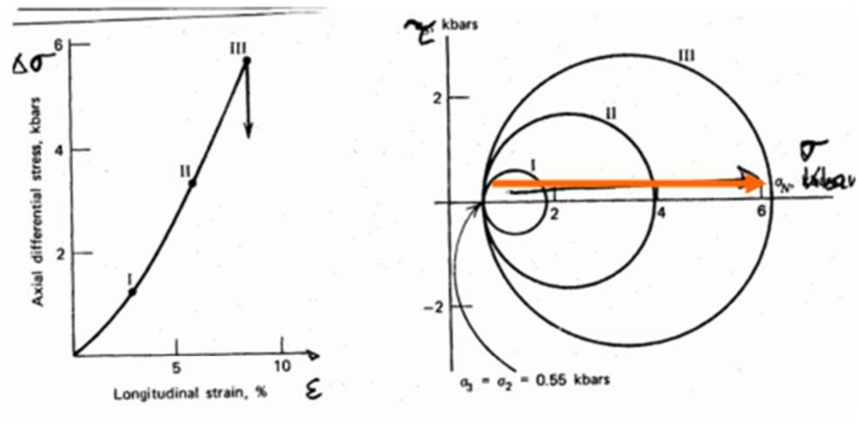


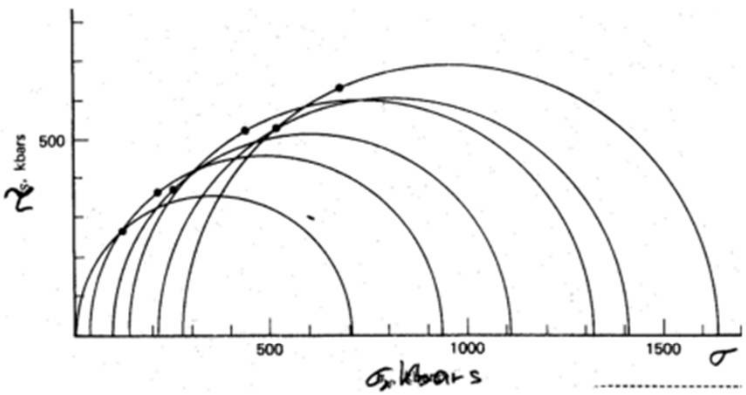


Shear Failure

Experiment monitored with the Mohr circle:

A critical circle is obtained which defines the conditions at which failure occurs

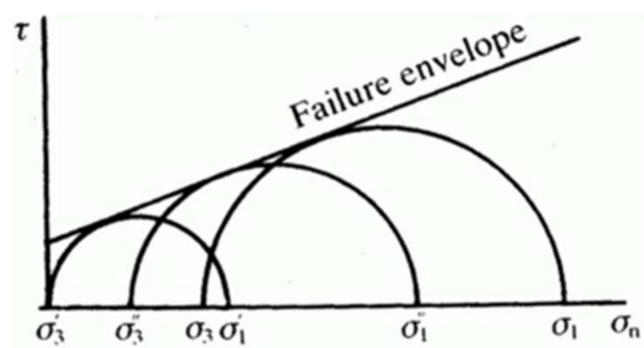




Repeating the same experiment under different confining pressures one obtains a series of critical circles

The failure envelope the stable field. This is the Mohr criterion

This is a purely empirical result based on the results of experiments

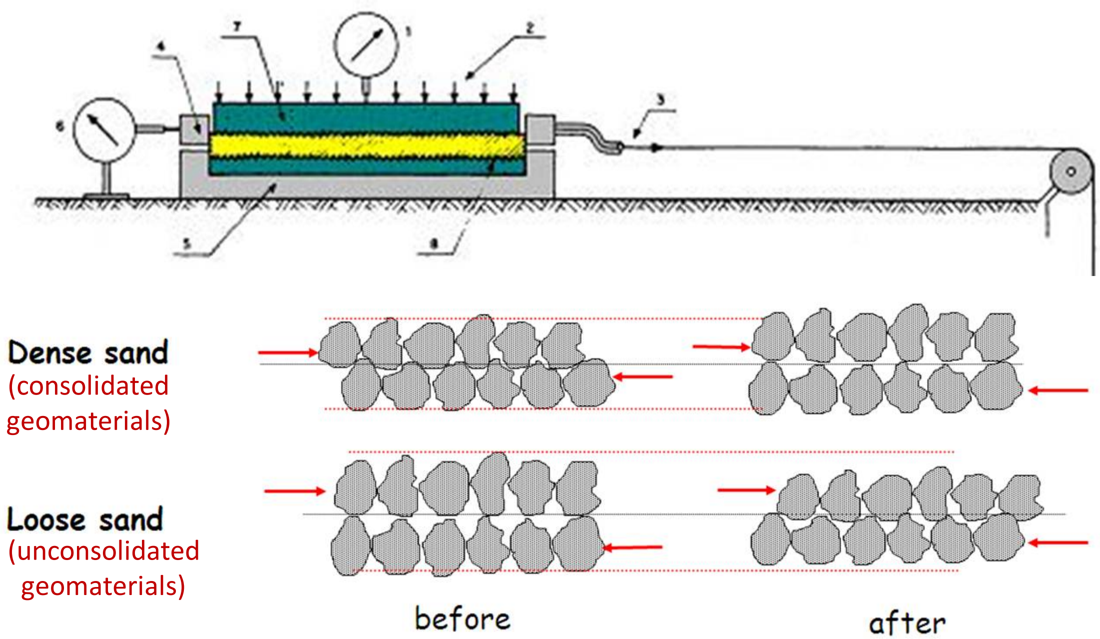


Rock Volumetric Behavior

Dilation **:**

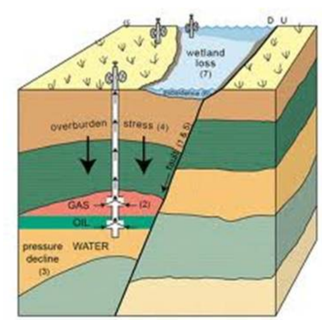
rock expansion under shear

Soil



Rock

Important for geological fault reactivation

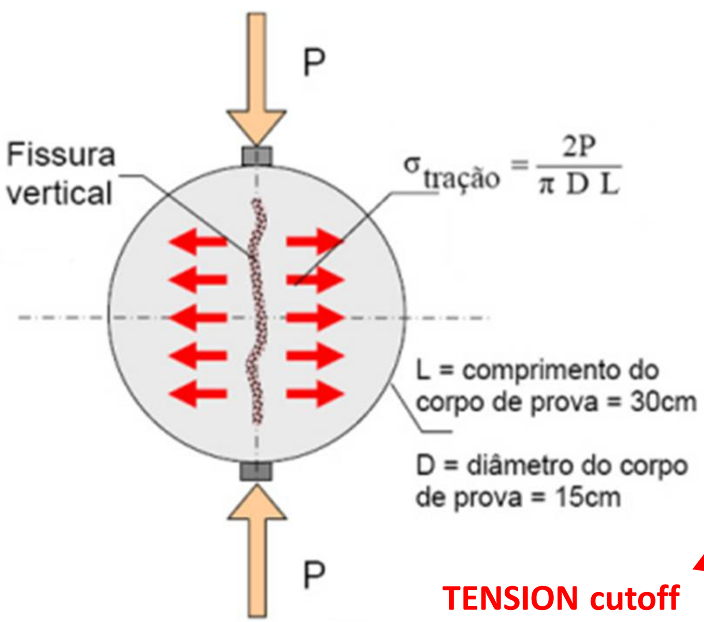


Tensile Failure

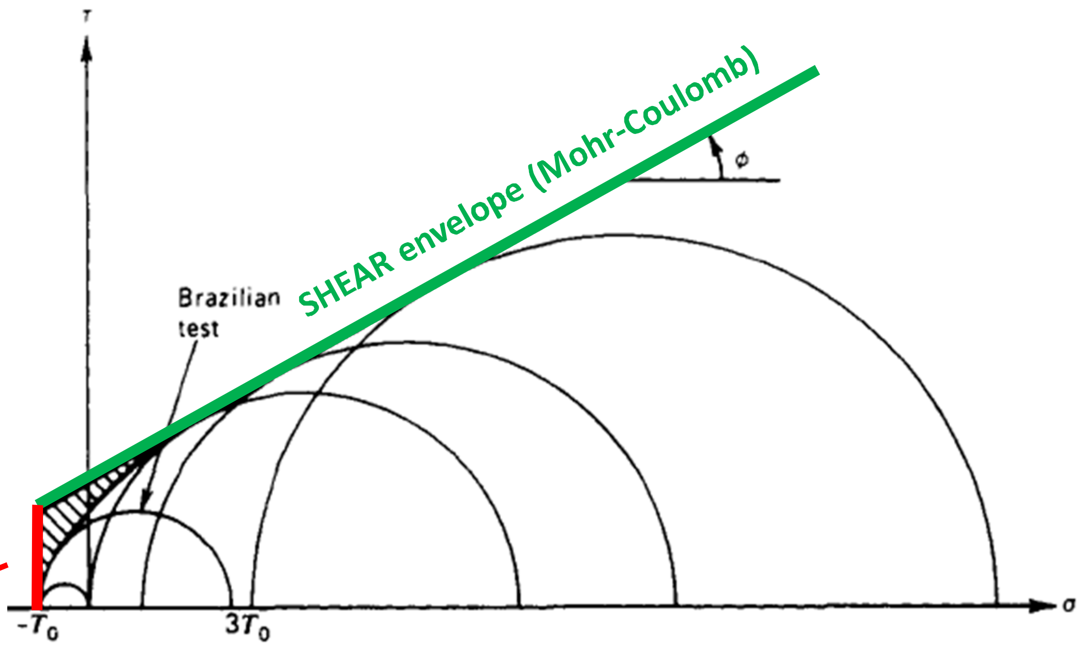
Tensile failure occurs when the **effective tensile stress** across some plane in the sample **exceeds a critical limit**. This limit is called the **tensile strength** (T_0) and has the same unit as stress. The tensile strength is a characteristic property of the rock. Most sedimentary rocks have a rather low tensile strength, typically only a few MPa or less. In fact, it is a standard approximation for several applications that the tensile strength is zero.
(Fjaer et al., 2008)

$$\sigma' \geq -T_0$$

- Brazilian Test:



- Incorporation of tensile strength in failure surface:

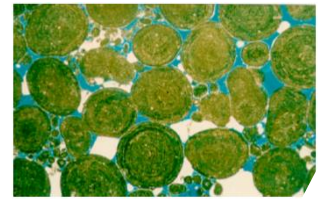


Rock Volumetric Behavior

UFPE

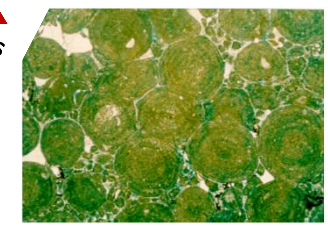
Rock compaction :

Volume deacrease due to compression



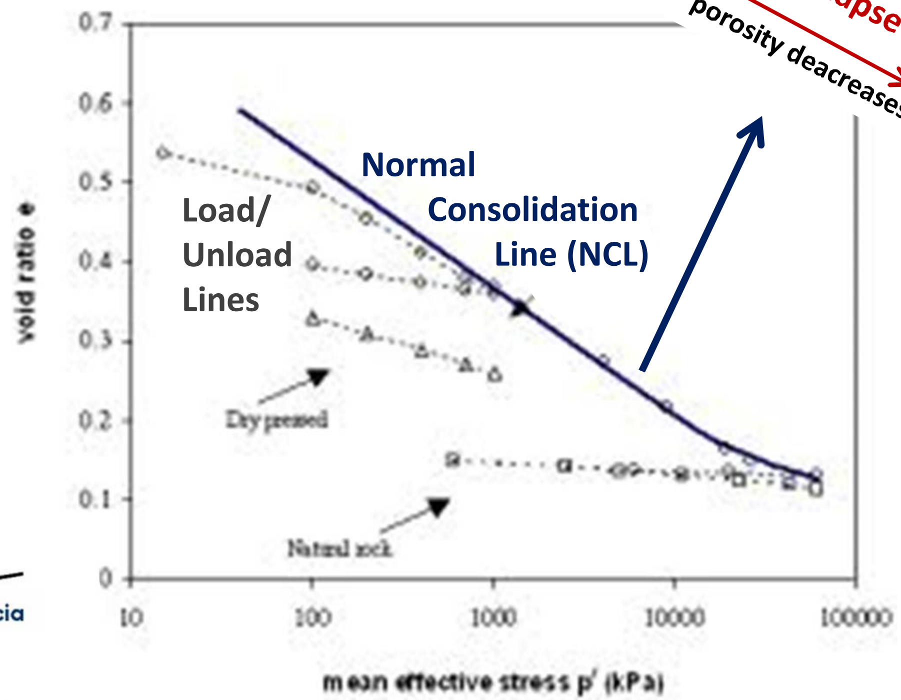
pore colapse

porosity deacreases

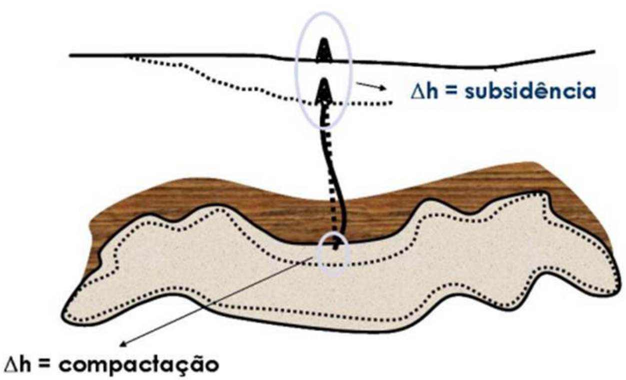


Normal Consolidation Line (NCL):
Change of pore structure
(LIMIT to plastic compaction:
irreversible behavior)

Load/Unload Line:
No changes in pore structure
(elastic deformation:
reversible behavior)

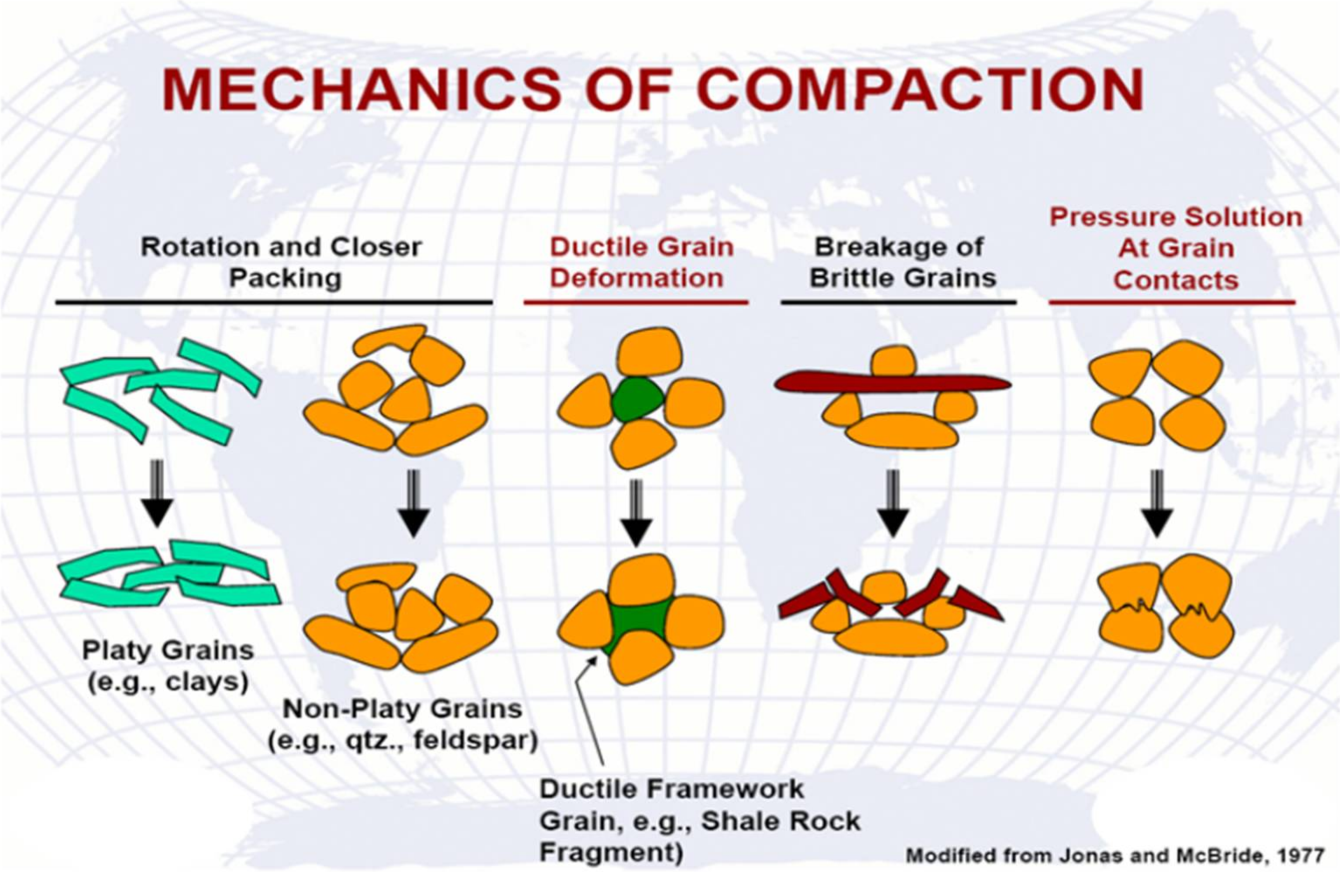


Isotropic compression of Bringelly shale



Important for reservoir compaction

Rock Volumetric Behavior



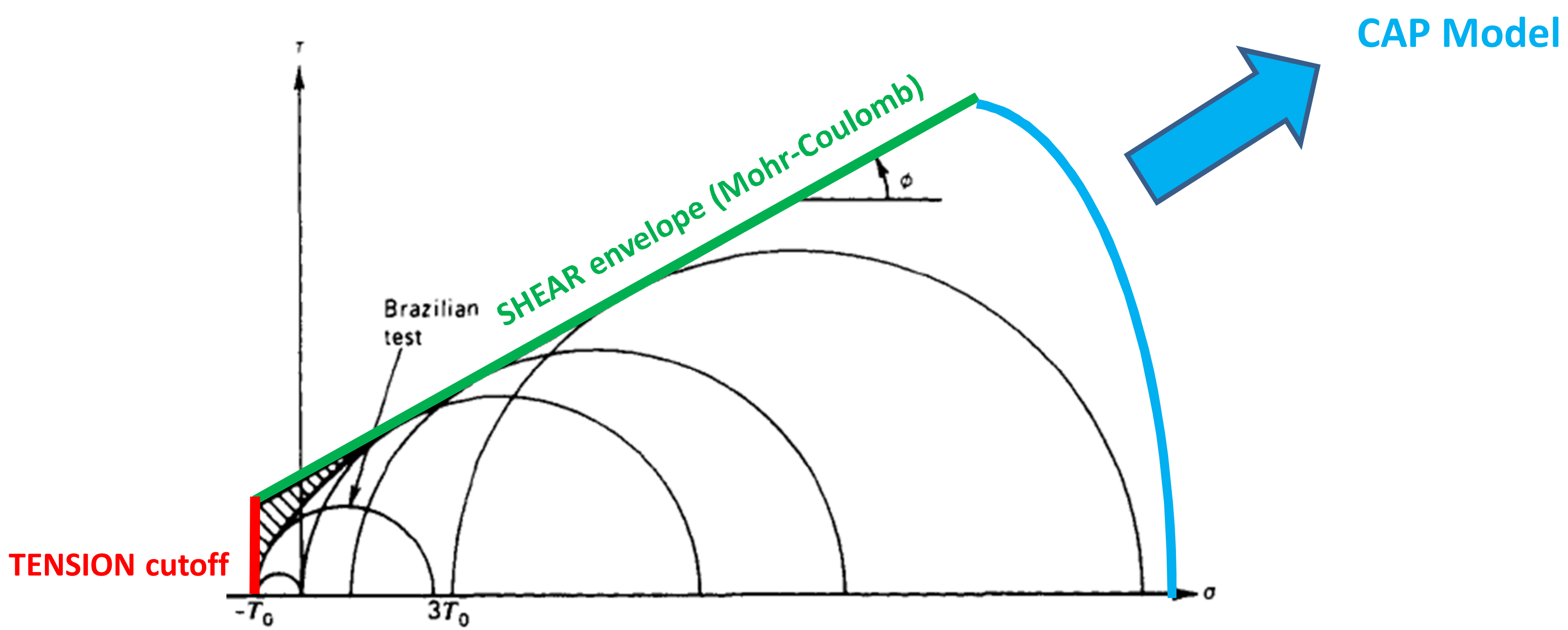
Irreversibility...

Rock Volumetric Behavior

Rock compaction **:**

Volume deacrease due to compression

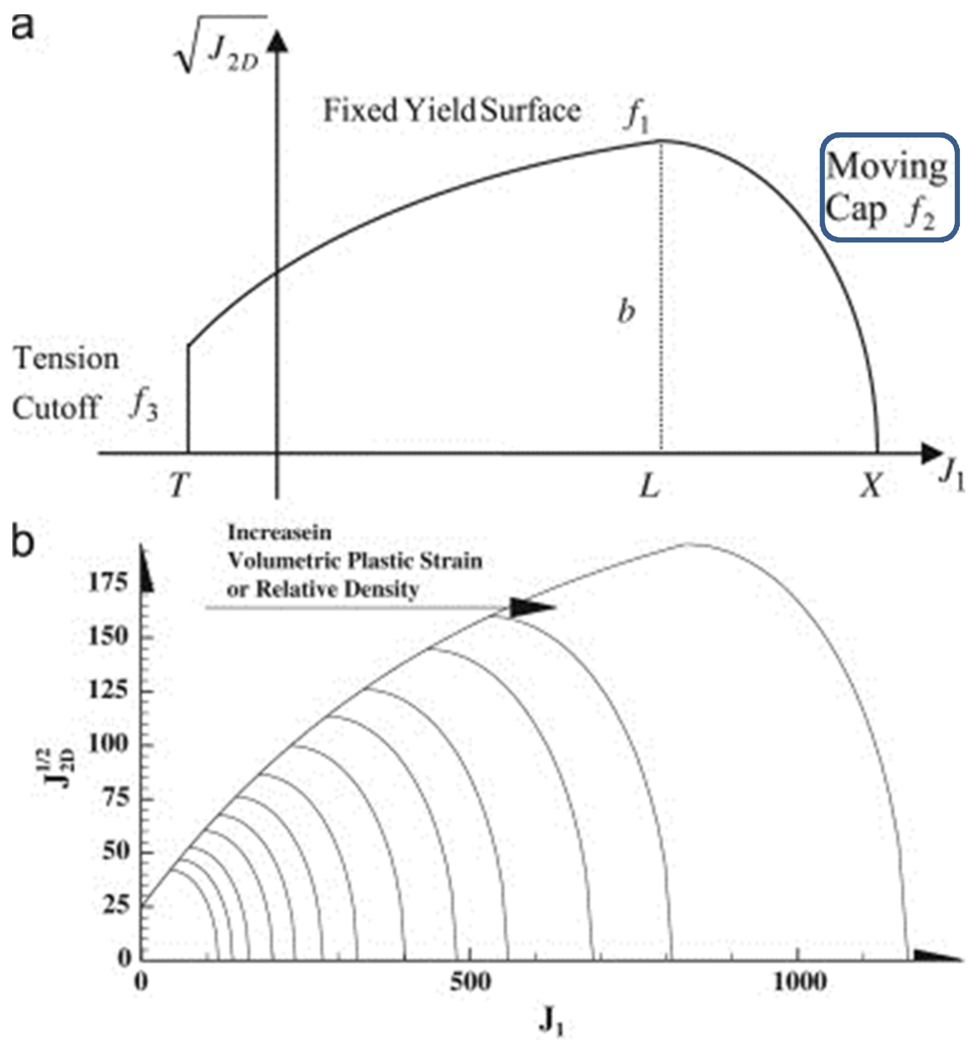
- **Incorporation of elastic-plastic compressive limit in failure surface:**



Rock Volumetric Behavior

CAP model:

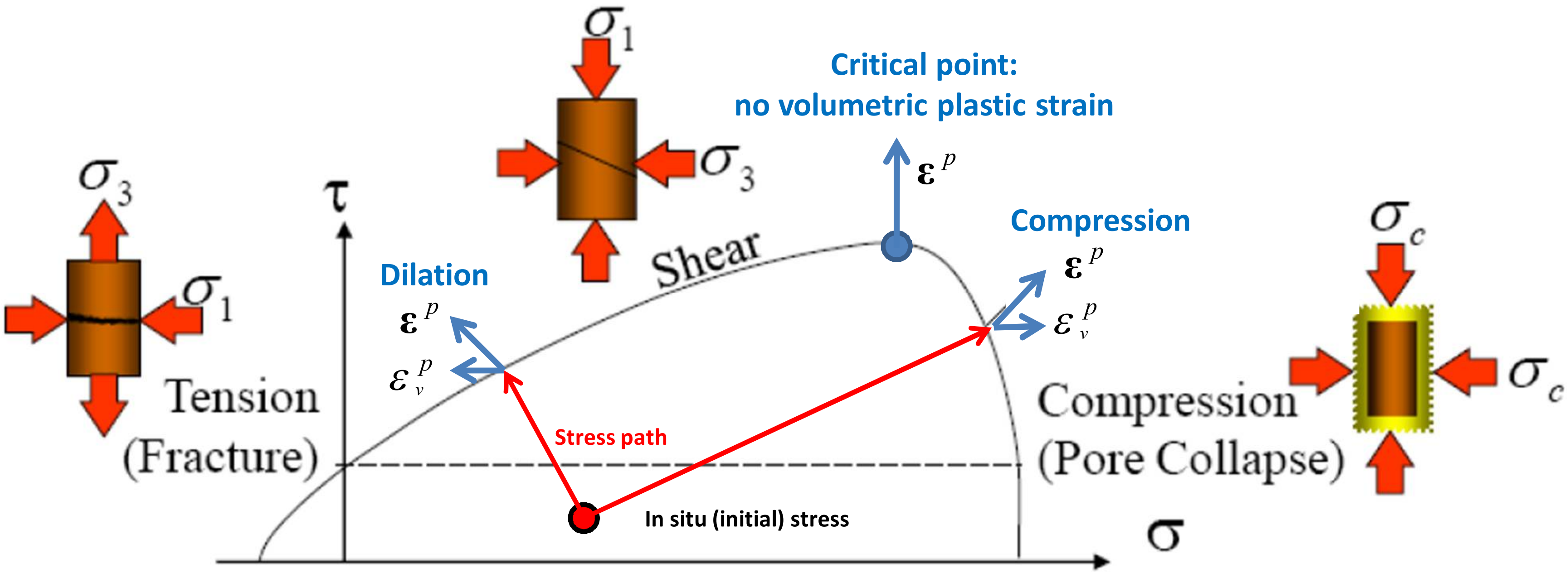
Multi-mechanism model



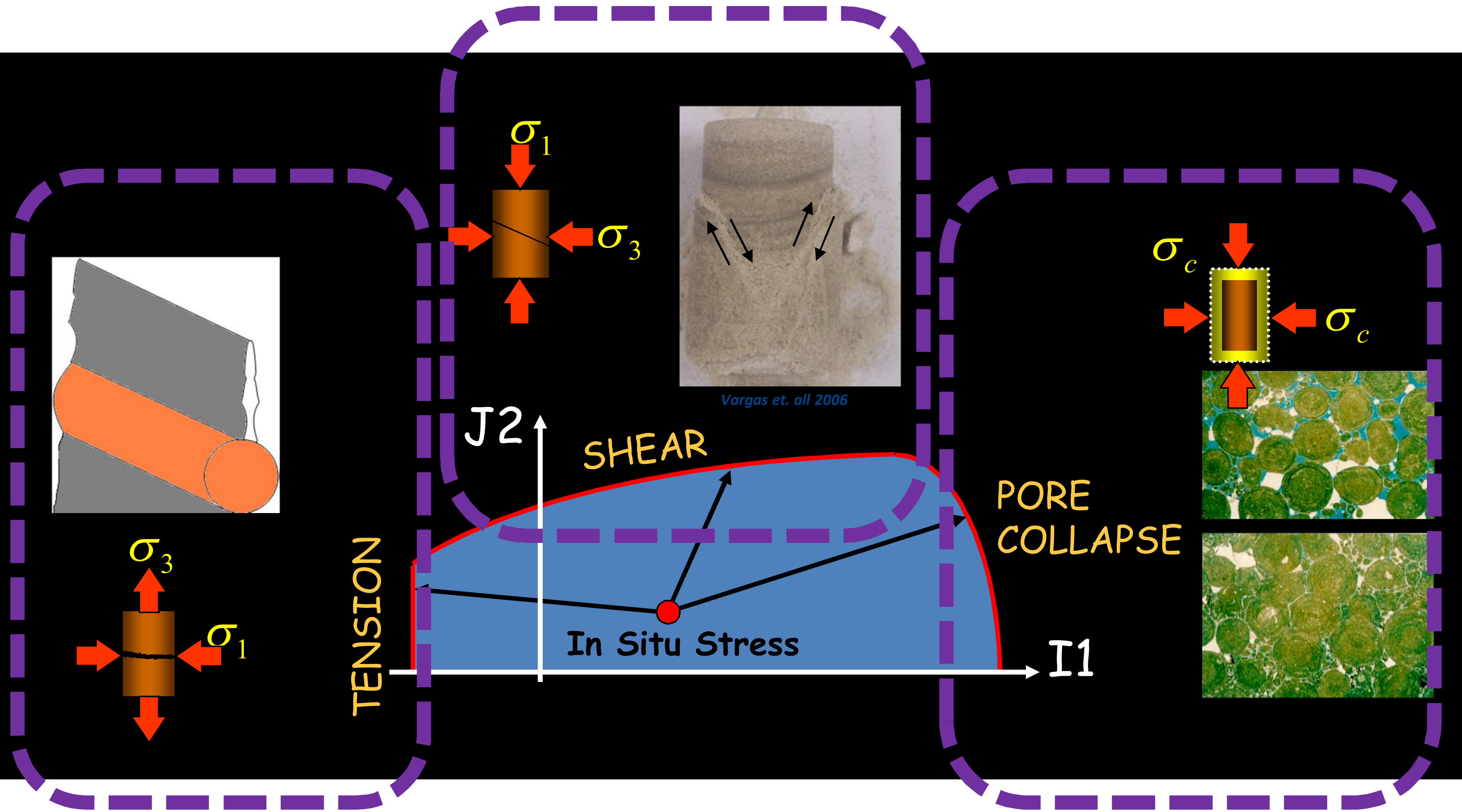
Rock Volumetric Behavior

CAP model:

Multi-mechanism model



FAULT REACTIVATION



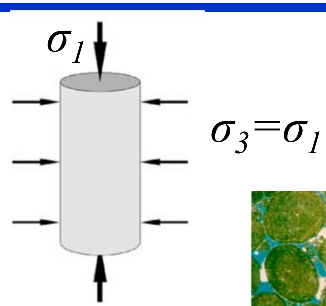
HYDRAULIC FRACTURING

RESERVOIR COMPACTION

Coupling Between Deformation and Permeability

Hydrostatic Compression:

Effect of reservoir compaction:
Decrease of permeability due to (irreversible) pore colapse.

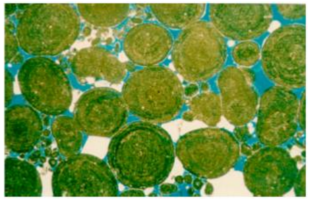


Ex: Kozeny-Carman

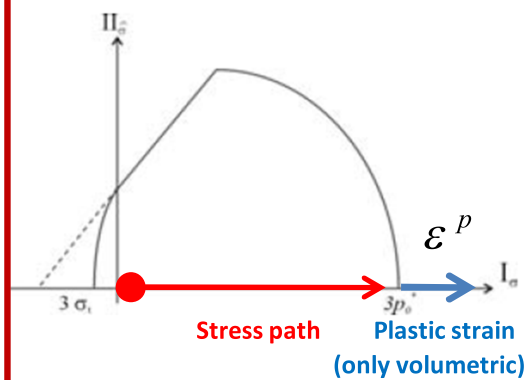
$$\mathbf{k} = \mathbf{k}_o \frac{\phi^3}{(1-\phi)^2} \frac{(1-\phi_o)^2}{\phi_o{}^3}$$

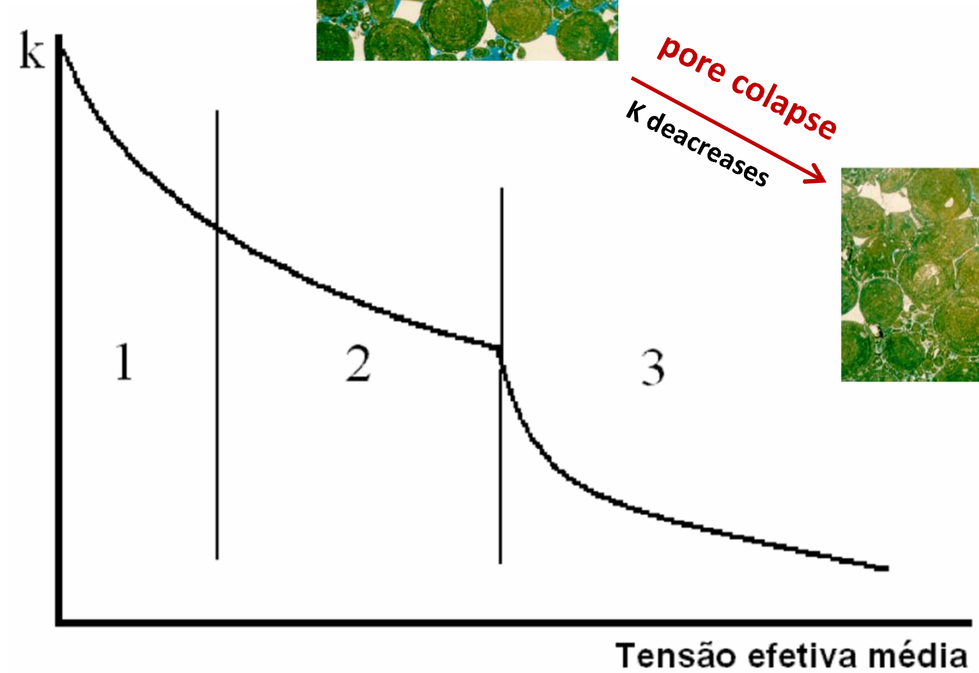
ϕ_o : reference porosity

$\mathbf{k}_o$: intrinsic permeability for matrix ϕ_o



Interpretation using CAP model





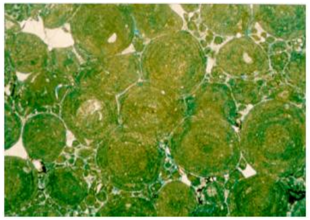
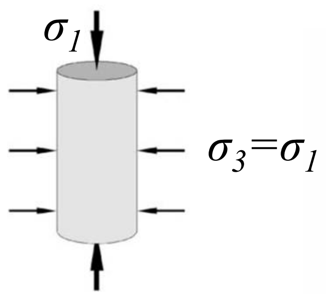


Figura 2.1: Esquema da variação da permeabilidade na compressão hidrostática (David *et al.* 1994). São observadas três fases: (1) fechamento de fissuras, (2) compactação dos poros e (3) esmagamento dos grãos.

Coupling Between Deformation and Permeability

Hydrostatic Compression:



Applied stress did not induce further anisotropy

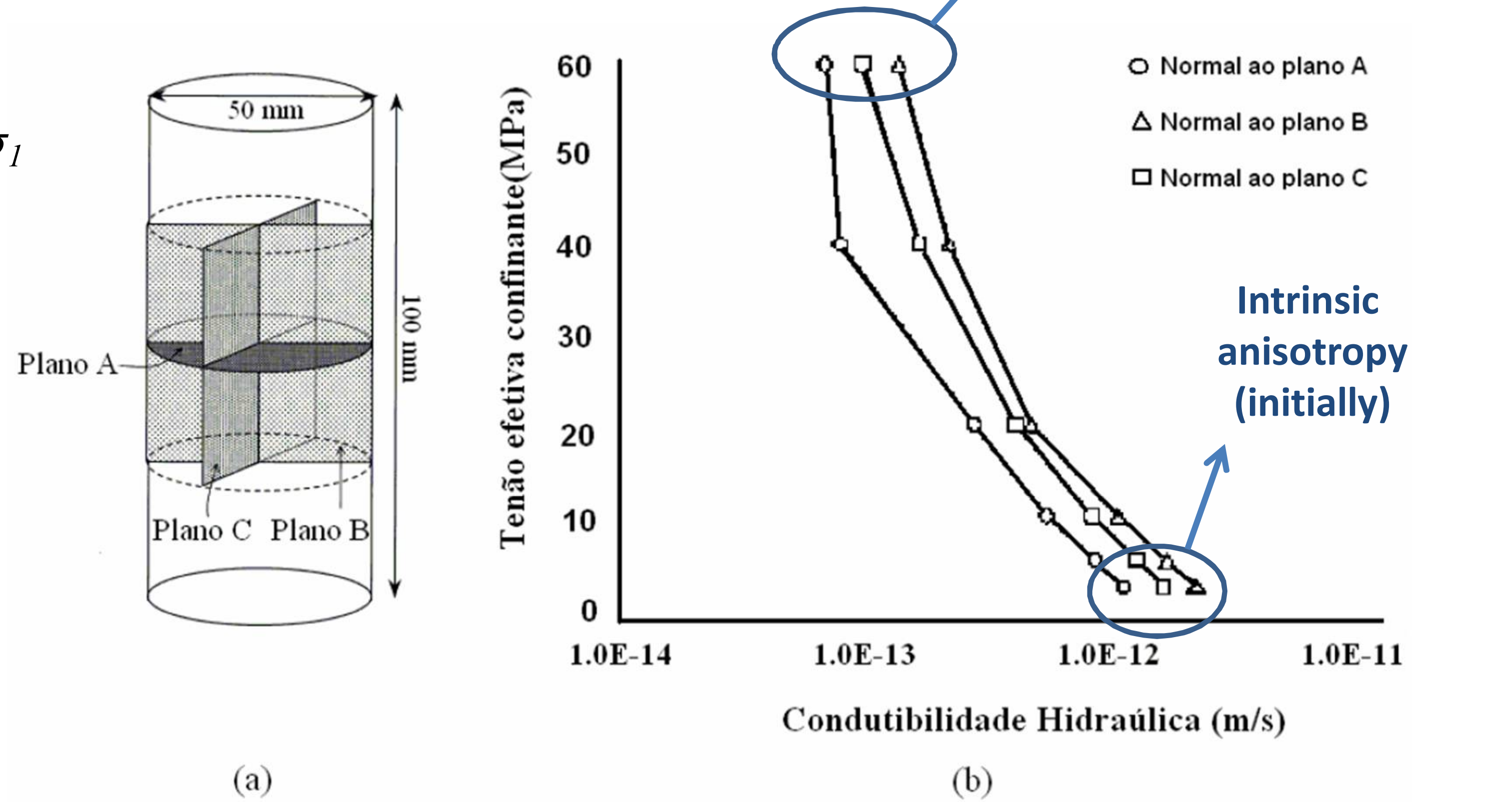
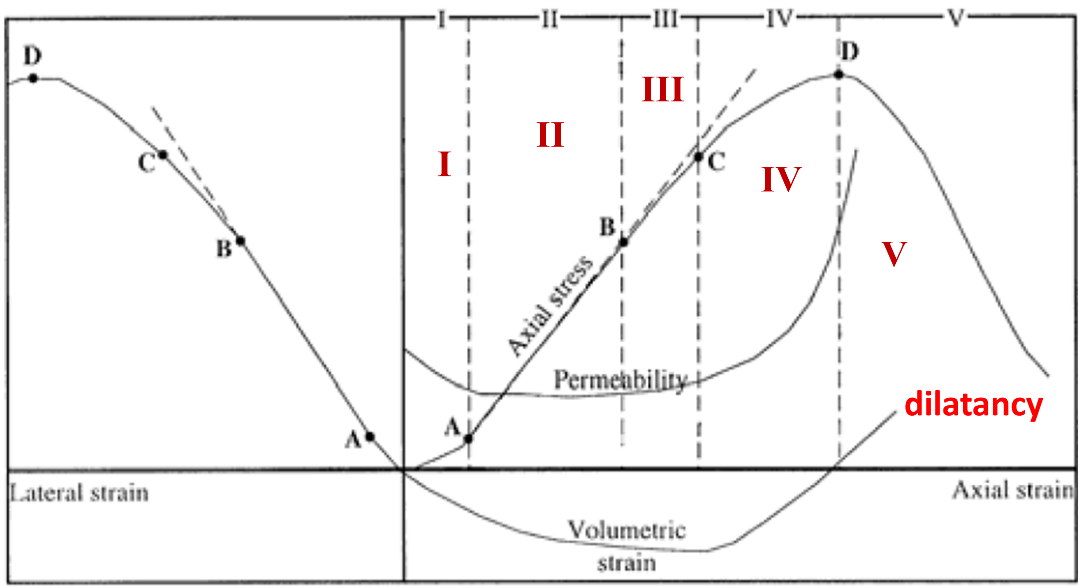


Figura 2.2: Ensaio de compressão hidrostática de Kiyama *et al.* (1996) para obtenção das permeabilidades em diferentes direções: (a) Planos considerados nas medições da permeabilidade; (b) Resultados em tensão confinante efetiva versus permeabilidade.

Coupling Between Deformation and Permeability

UFPE

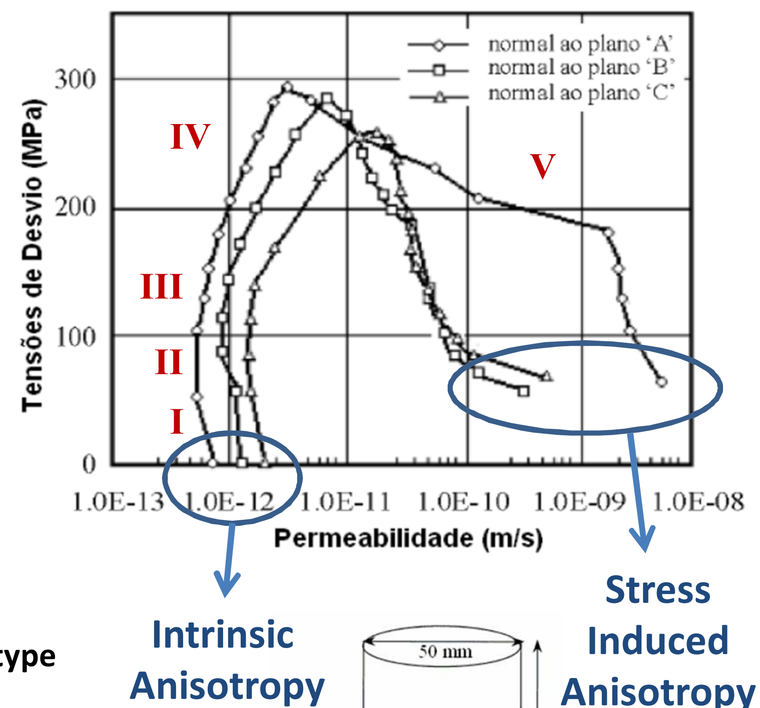
Triaxial Compression:



(adaptado de Paterson, 1978)

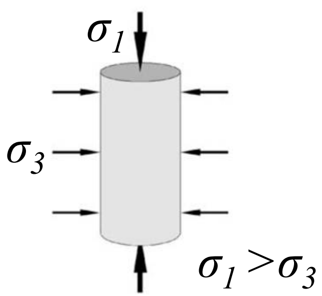
Anisotropy on permeability:

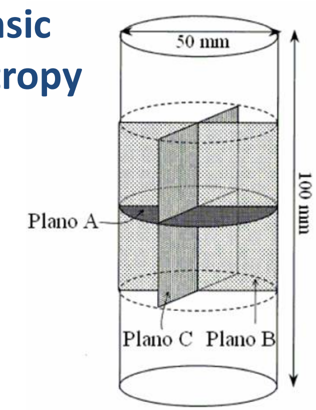
Kiyama et al. (1996)



Permeability variation in a brittle rock in a triaxial compression test. This type of stress-strain behavior is widely reported in the literature.
We have five distinct regions:

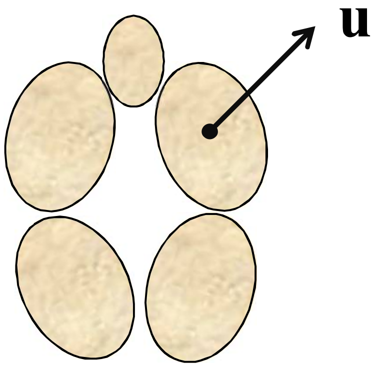
- **I - closure of pre-existing micro-cracks**
- **II - zone of elastic behavior**
- **III - steady growth of cracks**
- **IV - unstable growth of cracks**
- **V- post-peak zone characterized by the loss of resistance (softening followed by rupture) of the material**





Fluid Flow in Deformable Porous Media

$$\mathbf{u} = \begin{Bmatrix} u_x \\ u_y \\ u_z \end{Bmatrix}$$



$$\dot{\mathbf{u}} = \frac{d\mathbf{u}}{dt}$$

Volumetric strain:

$$\nabla \cdot \mathbf{u} = d\varepsilon_x + d\varepsilon_y + d\varepsilon_z$$

$$\nabla \cdot \frac{d\mathbf{u}}{dt} = \frac{d\varepsilon_x}{dt} + \frac{d\varepsilon_y}{dt} + \frac{d\varepsilon_z}{dt}$$

Fluid Flow in Deformable Porous Media

MASS BALANCE OF SOLID

Mass balance of solid present in the medium is written as:

$$\frac{\partial}{\partial t}\left(\rho_s\left(1-\phi\right)\right)+\nabla\cdot\left(\mathbf{j}_s\right)=0$$

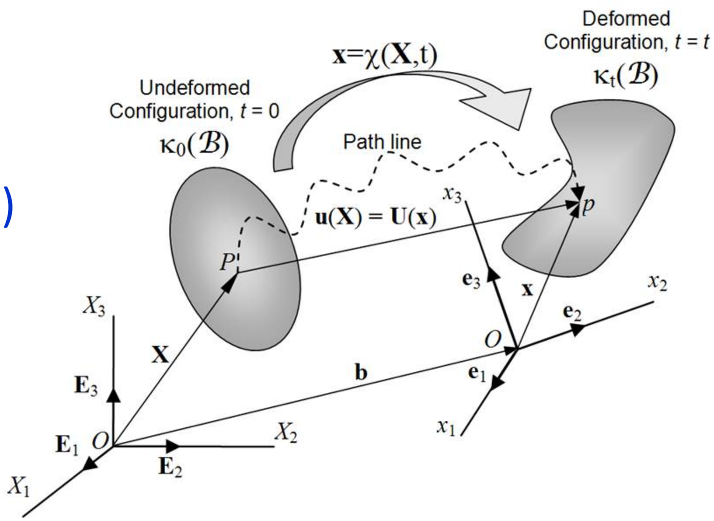
where ρ_s is the mass of solid per unit volume of solid and j_s is the flux of solid.

$$\frac{\partial}{\partial t}\left(\rho_s\left(1-\phi\right)\right)+\nabla\cdot\left(\rho_s\left(1-\phi\right)\dot{\mathbf{u}}\right)=0 \qquad (1)$$

$$\left(1-\phi\right)\frac{\partial\rho_s}{\partial t}-\rho_s\frac{\partial\phi}{\partial t}+\rho_s\left(1-\phi\right)\nabla\cdot\dot{\mathbf{u}}+\left(1-\phi\right)\dot{\mathbf{u}}\nabla\rho_s-\rho_s\dot{\mathbf{u}}\nabla\phi=0 \qquad (2)$$

MASS BALANCE OF SOLID

$$\frac{\partial}{\partial t}\left(\rho_s\left(1-\phi\right)\right)+\nabla\cdot\left(\rho_s\left(1-\phi\right)\dot{\mathbf{u}}\right)=0$$ (Eulerian description)



✓When the description of motion is made in terms of the spatial coordinates is called the spatial description or Eulerian description,

✓That is, the current configuration is taken as the reference configuration.

✓ In the Lagrangian description the position and physical properties of the particles are described in terms of the material or referential coordinates & time.

✓ The material derivative (or substantial time derivative) can serve as a link between 'Eulerian' and 'Lagrangian' descriptions of motion

✓A more convenient form of the balance equations is obtained considering the definitions of material derivate with respect to the solid velocity; which can be expressed generically as:

$$\frac{D(\circ)}{Dt}=\frac{\partial(\circ)}{\partial t}+\dot{\mathbf{u}}.\nabla(\circ) \qquad (3)$$

Fluid Flow in Deformable Porous Media

Definitions:

✓ Spatial point: fix point in the space

✓ Material point: a particle.

✓ The particle can be at different spatial points during its movement in time.

✓ Configuration (Ω): space occupied by the particles (that conform the continuum medium) at certain instant 't'

$$\mathbf{X} = X_1\hat{\mathbf{e}}_1 + X_2\hat{\mathbf{e}}_2 + X_3\hat{\mathbf{e}}_3 = X_i\hat{\mathbf{e}}_i$$

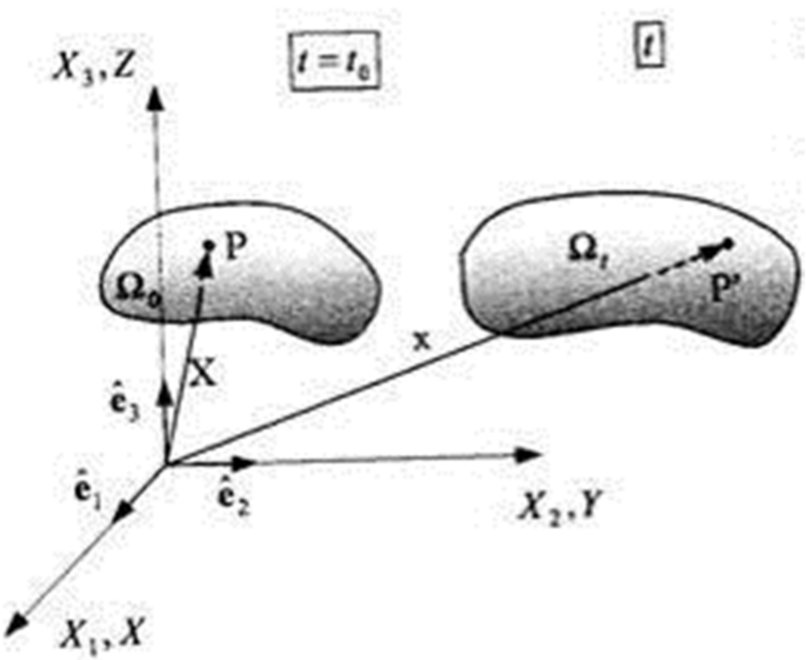
$t = t_o$ is the reference time

Ω_0 = initial material or reference configuration.

Ω_t = current configuration

Material coordinates (X_1, X_2, X_3)

Spatial coordinates (x_1, x_2, x_3) (current configuration).



Fluid Flow in Deformable Porous Media

✓ The movement of the particles (which conform the continuum medium) can be described by the evolution of their spatial coordinates (or their 'position vector') in time.

✓ We need to know a function for each particle (identify by a 'label'), which provide the spatial coordinates x_i (or the corresponding vector) in the successive instants of time.

✓ As a label, to characterize unequivocally each particle, it is possible to use the 'material coordinates'.

✓In this manner, the 'movement equations' are obtained:

$$\mathbf{x} = \varphi\ (\text{partícula}, t) = \varphi(\mathbf{X}, t) \overset{not}{=} \mathbf{x}(\mathbf{X}, t)$$

$$x_i = \varphi_i(X_1, X_2, X_3, t) \quad i \in \{1,2,3\}$$

✓Which provide the spatial coordinates as a function of the material ones.

✓The 'inverse movement equations' are given by:

$$\mathbf{X} = \varphi^{-1}(\mathbf{x}, t) \overset{not}{=} \mathbf{X}(\mathbf{x}, t)$$

$$X_i = \varphi_i^{-1}(x_1, x_2, x_3, t) \quad i \in \{1,2,3\}$$

✓Which provide the material coordinates as a function of the spatial ones.

➢Description of the movement

✓Material description: A property is described (i.e. density ρ) using as argument the material coordinate.

$$\rho = \overline{\rho}(\mathbf{X},t) = \overline{\rho}(X_1, X_2, X_3, t)$$

✓Note that if we fix $X=(X_1, X_2, X_3)$, we are following the density variation specific particle.

✓Because of that the name of 'material description'(Lagrangian description).

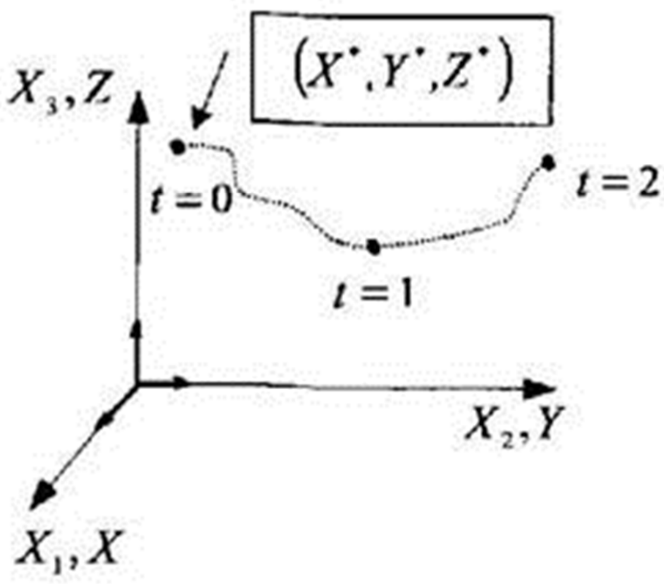
✓Spatial description: A property is described (i.e. density ρ) using as argument the spatial coordinate (Eulerian description).

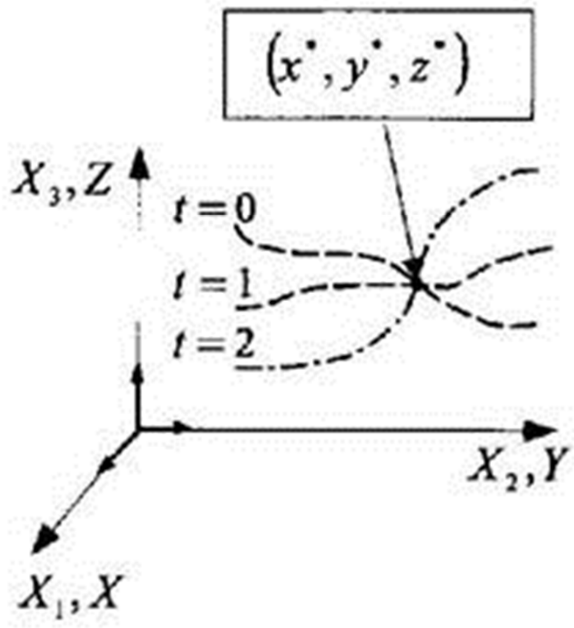
$$\rho = \rho(\mathbf{x},t) = \rho(x_1, x_2, x_3, t)$$

✓Note that if we fix $x=(x_1, x_2, x_3)$, we focus the attention on one point of the space; and we follow the density evolution for the different particles that are passing for this fix spatial point.

Fluid Flow in Deformable Porous Media

$$\rho(\mathbf{x},t) = \rho(\mathbf{x}(\mathbf{X},t),t) = \bar{\rho}(\mathbf{X},t)$$
$$\bar{\rho}(\mathbf{X},t) = \bar{\rho}(\mathbf{X}(\mathbf{x},t),t) = \rho(\mathbf{x},t) \qquad (1.9)$$





Olivella and Argelet (2000)

Temporal, local, material and convective derivative.

✓Consider a given property and their respective descriptions material and spatial.

$$\Gamma(\mathbf{X},t)=\gamma(\mathbf{x},t)$$

✓ We pass from one description to the other by using the 'movement equations'

✓Local derivative: it is the variation of a property in time of a fix point in the space.

✓It is possible to write

local derivative $\overset{not}{=} \dfrac{\partial\gamma(\mathbf{x},t)}{\partial t}$

✓Material derivative: it is the variation of a property in time following a specific particle (material point) of the continuum medium.

✓ It is possible to write this derivative as:

material derivative $\overset{not}{=} \dfrac{d}{dt}\Gamma = \dfrac{\partial\Gamma(\mathbf{X},t)}{\partial t}$ *also denoted as :* $\dfrac{D\,\Gamma}{Dt}$

Fluid Flow in Deformable Porous Media

✓If we start with the spatial description of the property and we consider implicit in this equation the 'movement equation':

$$\gamma(\mathbf{x},t)=\gamma(\mathbf{x}(\mathbf{X},t),t)=\Gamma(\mathbf{X},t)$$

✓We can obtain the material derivative (i.e. following the particle) from spatial description:

material derivative $\overset{\text{not}}{=} \dfrac{d}{dt}\gamma(\mathbf{x}(\mathbf{X},t),t)=\dfrac{\partial\Gamma(\mathbf{X},t)}{\partial t}$

✓Velocity is the derivative of movement equations respect to time:

$$\frac{\partial \mathbf{x}(\mathbf{X},t)}{\partial t}=\mathbf{V}(\mathbf{X}(\mathbf{x},t),t)=\mathbf{v}(\mathbf{x},t)$$

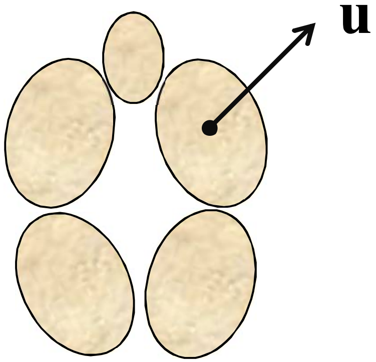
✓Finally:

$$\frac{d\gamma(\mathbf{x}(\mathbf{X},t),t)}{dt}=\frac{\partial\gamma(\mathbf{x},t)}{\partial t}+\frac{\partial\gamma}{\partial x_i}\frac{\partial x_i}{\partial t}=\frac{\partial\gamma(\mathbf{x},t)}{\partial t}+\frac{\partial\gamma}{\partial\mathbf{x}}\cdot\underbrace{\frac{\partial\mathbf{x}}{\partial t}}_{\mathbf{v}(\mathbf{x},t)}$$

✓We can generalized that definition for any property (scalar or vectorial):

$$\underbrace{\frac{D\chi}{Dt}=\frac{d\chi(\mathbf{x},t)}{dt}}_{\text{material derivative}}=\underbrace{\frac{\partial\chi(\mathbf{x},t)}{\partial t}}_{\text{local derivative}}+\underbrace{\mathbf{v}(\mathbf{x},t)\cdot\nabla\chi(\mathbf{x},t)}_{\text{convective derivative}}$$

$$\mathbf{u} = \begin{Bmatrix} u_x \\ u_y \\ u_z \end{Bmatrix}$$



$$\dot{\mathbf{u}} = \frac{d\mathbf{u}}{dt}$$

Volumetric strain:

$$\nabla \cdot \mathbf{u} = d\varepsilon_x + d\varepsilon_y + d\varepsilon_z$$

$$\nabla \cdot \frac{d\mathbf{u}}{dt} = \frac{d\varepsilon_x}{dt} + \frac{d\varepsilon_y}{dt} + \frac{d\varepsilon_z}{dt}$$

UFPE

MASS BALANCE OF SOLID

$$\frac{\partial}{\partial t}\left(\rho_s\left(1-\phi\right)\right)+\nabla\cdot\left(\rho_s\left(1-\phi\right)\dot{\mathbf{u}}\right)=0$$

(Eulerian description)

$$\frac{D(\circ)}{Dt}=\frac{\partial(\circ)}{\partial t}+\dot{\mathbf{u}}.\nabla(\circ)$$

(Material derivative)

$$\left(1-\phi\right)\frac{\partial\rho_s}{\partial t}-\rho_s\frac{\partial\phi}{\partial t}+\rho_s\left(1-\phi\right)\nabla\cdot\dot{\mathbf{u}}+\left(1-\phi\right)\dot{\mathbf{u}}\,\nabla\rho_s-\rho_s\,\dot{\mathbf{u}}\,\nabla\phi=0$$

(equation 2)

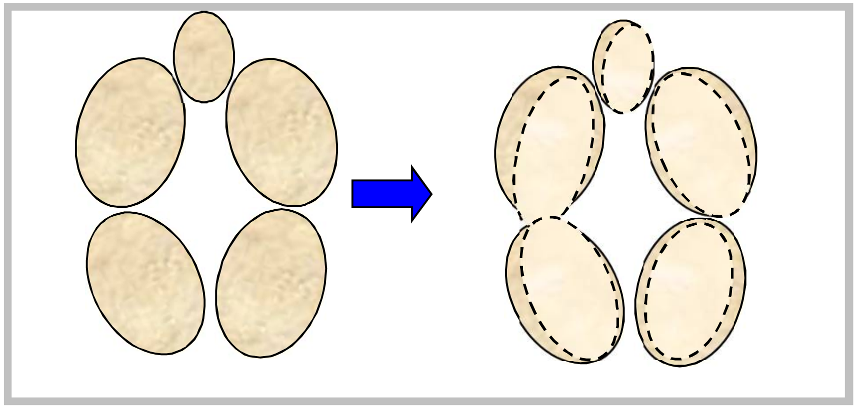
$$\rho_s\left(1-\phi\right)\nabla\cdot\dot{\mathbf{u}}+\underbrace{\left(1-\phi\right)\frac{\partial\rho_s}{\partial t}+\left(1-\phi\right)\dot{\mathbf{u}}\,\nabla\rho_s}_{\left(1-\phi\right)\frac{D\rho_s}{Dt}}\underbrace{-\rho_s\frac{\partial\phi}{\partial t}+-\rho_s\,\dot{\mathbf{u}}\,\nabla\phi}_{-\rho_s\frac{D\phi}{Dt}}=0$$

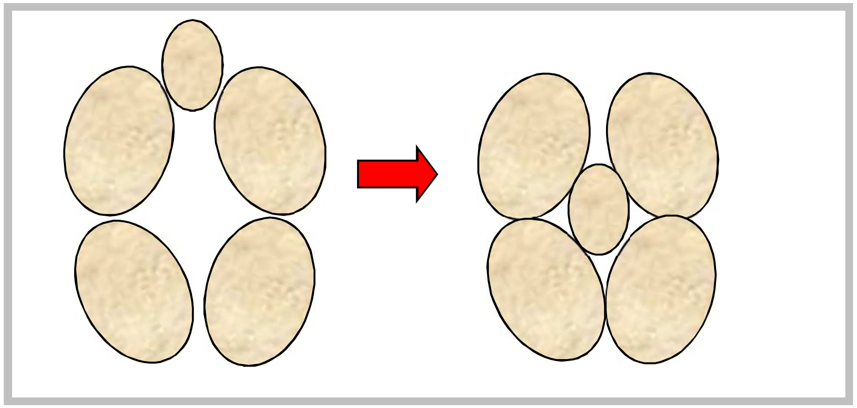
$$\frac{D\phi}{Dt}=\frac{\left(1-\phi\right)}{\rho_s}\frac{D\rho_s}{Dt}+\left(1-\phi\right)\nabla\cdot\dot{\mathbf{u}} \qquad (4)$$

(Lagrangian description)

Fluid Flow in Deformable Porous Media

$$\frac{D_s\phi}{Dt} = \frac{1}{\rho_s}\left[(1-\phi)\frac{D_s\rho_s}{Dt}\right] + (1-\phi)\nabla\cdot\frac{d\mathbf{u}}{dt}$$





Detailed description of solid density variation (including rock compressibility C_r) can be found in Lewis and Schrefler (1998).

Volumetric strain:

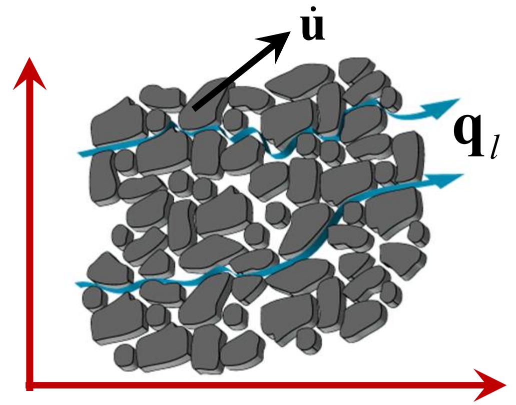
$$\nabla\cdot\frac{d\mathbf{u}}{dt} = \frac{d\varepsilon_x}{dt} + \frac{d\varepsilon_y}{dt} + \frac{d\varepsilon_z}{dt}$$

❑ Flow Equation – Water Saturated Porous Media

➢ Water Mass Balance Equation

$$\frac{\partial}{\partial t}\left(\rho_l \phi\right)+\nabla\cdot\left(\rho_l \mathbf{q}_l+\rho_l \phi \dot{\mathbf{u}}\right)=0$$

$\mathbf{q}_l$: velocity respect to the solid skeleton



This term considers the velocity of the liquid respect to the solid skeleton (q_l) + the velocity of the solid respect to a fix reference system $(\dot{\mathbf{u}})$. This is because the solid is moving now and drag the liquid phase with it.

$$\phi\frac{\partial \rho_l}{\partial t}+\rho_l\frac{\partial \phi}{\partial t}+\nabla\rho_l.\mathbf{q}_l+\rho_l\nabla.\mathbf{q}_l+\phi\dot{\mathbf{u}}.\nabla\rho_l+\rho_l\dot{\mathbf{u}}.\nabla\phi+\rho_l\phi\nabla.\dot{\mathbf{u}}=0$$

Using (3)

$$\phi\frac{D_s\rho_l}{Dt}+\rho_l\frac{D_s\phi}{Dt}+\nabla\rho_l.\mathbf{q}_l+\rho_l\nabla.\mathbf{q}_l+\rho_l\phi\nabla.\dot{\mathbf{u}}=0$$

Replacing (4) i.e. solid mass balance in material description

$$\frac{D\phi}{Dt}=\frac{(1-\phi)}{\rho_s}\frac{D\rho_s}{Dt}+(1-\phi)\nabla\cdot\dot{\mathbf{u}}$$

$$\phi\frac{D_s\rho_l}{Dt}+\rho_l\frac{(1-\phi)}{\rho_s}\frac{D\rho_s}{Dt}+\rho_l\cancel{(1-\phi)\nabla\cdot\dot{\mathbf{u}}}+\nabla\rho_l.\mathbf{q}_l+\rho_l\nabla.\mathbf{q}_l+\cancel{\rho_l\phi\nabla.\dot{\mathbf{u}}}=0$$

Flow Equation – Water Saturated Porous Media

Water Mass Balance Equation

$$\phi \frac{D_s \rho_l}{Dt} + (1-\phi)\frac{\rho_l}{\rho_s}\frac{D_s \rho_s}{Dt} + \rho_l \nabla \cdot \dot{\mathbf{u}} + \nabla \rho_l . \mathbf{q}_l + \rho_l \nabla . \mathbf{q}_l = 0$$

$$\phi \frac{D_s \rho_l}{Dt} + (1-\phi)\frac{\rho_l}{\rho_s}\frac{D_s \rho_s}{Dt} + \rho_l \nabla \cdot \dot{\mathbf{u}} + \nabla . (\rho_l \mathbf{q}_l) = 0$$

(Lagrangian description Mass Balance of Water)

- If there is a source or sink of water

$$\phi \frac{D_s \rho_l}{Dt} + (1-\phi)\frac{\rho_l}{\rho_s}\frac{D_s \rho_s}{Dt} + \rho_l \nabla \cdot \dot{\mathbf{u}} + \nabla . (\rho_l \mathbf{q}_l) = f_l$$

- If solid density is constant

$$\phi \frac{D_s \rho_l}{Dt} + \rho_l \nabla \cdot \dot{\mathbf{u}} + \nabla . (\rho_l \mathbf{q}_l) = 0$$

- If liquid density is constant

$$\nabla \cdot \dot{\mathbf{u}} + \nabla . \mathbf{q}_l = 0$$

SUMMARY

HM FORMULATION

Mechanical problem for geomaterials:

- **Equilibrium Equation:**

$$\nabla \cdot \boldsymbol{\sigma} + \mathbf{b} = \mathbf{0}$$

- **Principle of Effective Stresses:**

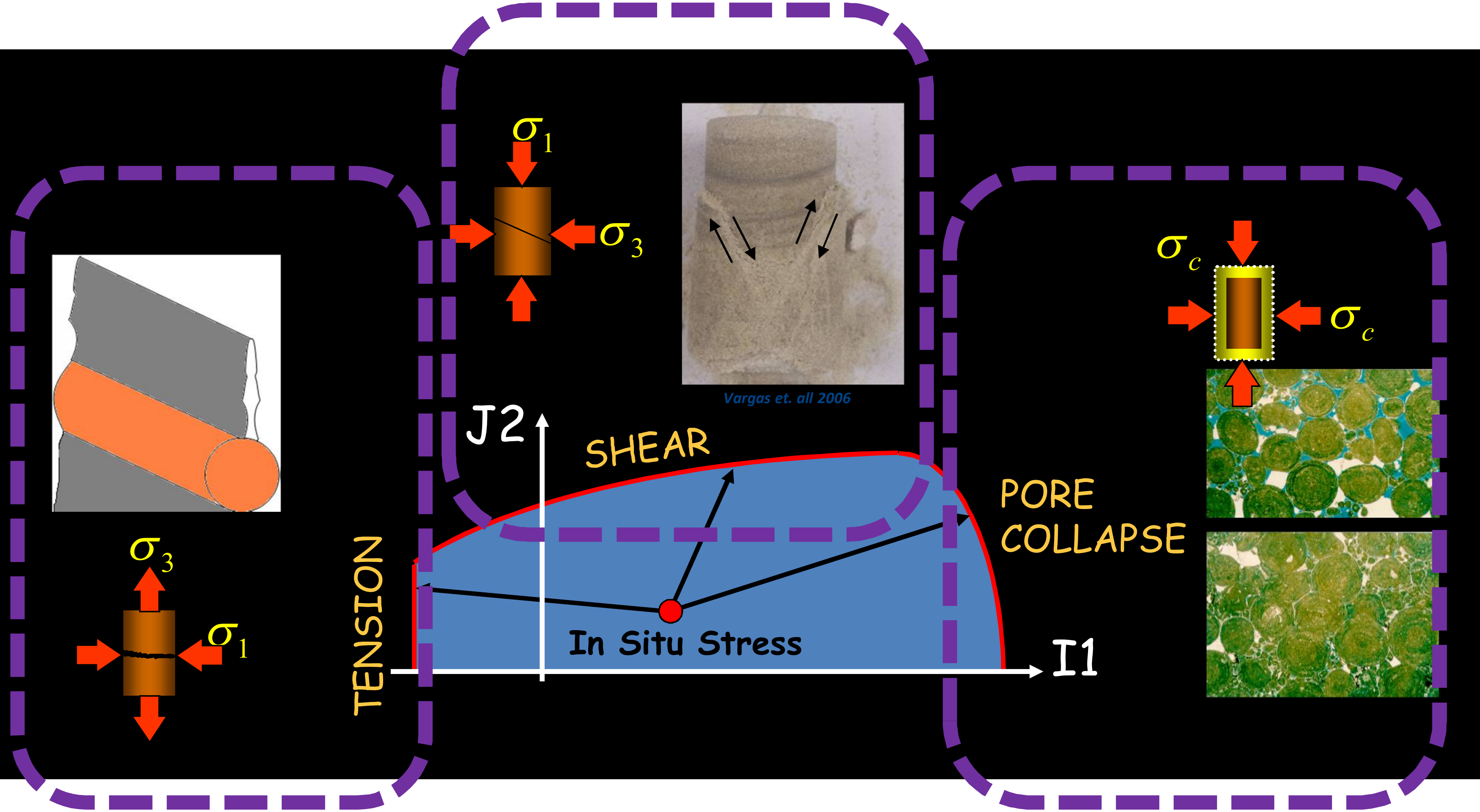
$$\boldsymbol{\sigma} = \boldsymbol{\sigma}' + \alpha \cdot p_f \cdot \mathbf{I}$$

- **Stress-strain relationship:**

$$\mathbf{d}\boldsymbol{\sigma}' = \mathbf{D} \cdot \mathbf{d}\boldsymbol{\varepsilon}$$

Specific for each geomaterial

FAULT REACTIVATION



HYDRAULIC FRACTURING

RESERVOIR COMPACTION

HM FORMULATION

HYDRO-MECHANICAL COUPLINGS:

- Rock porosity:

$$\frac{\partial}{\partial t}\left[(1-\phi)\rho_s\right]+\nabla\left[(1-\phi)\rho_s\,.\dot{\mathbf{u}}\right]=0$$

(mass conservation of solids)

(material derivative)

$$\frac{d\,\bullet}{dt}=\frac{\partial\,\bullet}{\partial t}+\dot{\mathbf{u}}\cdot\nabla\,\bullet$$

$$\frac{d\phi}{dt}=\frac{(1-\phi)}{\rho_s}\frac{d\rho_s}{dt}+(1-\phi)\frac{d\varepsilon_v}{dt}$$

(porosity update)

$$\frac{d\varepsilon_v}{dt}=\dot{\varepsilon}_v=\nabla\cdot\dot{\mathbf{u}}$$

- Rock permeability:

$$\mathbf{k}=\mathbf{k_i}\,\exp\left[b(\phi-\phi_i)\right]$$

Other: Kozeny-Carman

$$\mathbf{k}=\mathbf{k}_o\,\frac{\phi^3}{(1-\phi)^2}\,\frac{(1-\phi_o)^2}{\phi_o{}^3}$$

ϕ_o : reference porosity

$\mathbf{k}_o$: intrinsic permeability for matrix ϕ_o

HM FORMULATION

Hydraulic problem: two phase flow equations for deformable porous media

$$\phi \frac{D_s S_\alpha \rho_\alpha}{Dt} + (1-\phi)\frac{S_\alpha \rho_\alpha}{\rho_s}\frac{D_s \rho_s}{Dt} + S_\alpha \rho_\alpha \nabla \cdot \dot{\mathbf{u}} + \nabla \cdot (\rho_\alpha \mathbf{q}_\alpha) = 0 \qquad \alpha = w, o$$

$$\mathbf{q}_\alpha = -\frac{\mathbf{k} k_{r\alpha}}{\mu_\alpha}(\nabla p_\alpha - \rho_\alpha \mathbf{g})$$

where: $S_w + S_o = 1$ $\quad p_c = p_o - p_w$ $\quad \lambda_\alpha = k_{r\alpha}/\mu_\alpha$

ϕ	porosity
s_α	fluid saturation
ρ_α	fluid density
q_α	Darcy flow
μ_α	phase viscosity
$\dot{\mathbf{u}}$	Solid velocity
$\mathbf{k}$	permeability tensor
$k_{r\alpha}$	fluid relative permeability
λ_α	fluid mobility
p_α	Fluid pressure
p_c	capillary pressure
$\tilde{g}$	gravity

APPLICATION:

Primary Recovery – Reservoir Depletion

Increase of fluid flow apparent velocity

$$\mathbf{v}_\alpha = \frac{\mathbf{q}_\alpha}{\phi}$$

Porosity decrease

$$\frac{D}{Dt}\phi = \frac{(1-\phi)}{\rho_s}\frac{D}{Dt}\rho_s + (1-\phi)\nabla\cdot\dot{\mathbf{u}}$$

Volumetric strain in mass balance equations:

Compaction-driven mechanism

Later...

Pressure maintenance

$$\phi\frac{D_s S_\alpha \rho_\alpha}{Dt} + (1-\phi)\frac{S_\alpha \rho_\alpha}{\rho_s}\frac{D_s \rho_s}{Dt} + S_\alpha \rho_\alpha \nabla\cdot\dot{\mathbf{u}} + \nabla\cdot\left(\rho_\alpha \mathbf{q}_\alpha\right) = 0 \qquad \alpha = w, o$$

HM FORMULATION

HYDRAULIC

- **Water and oil flow: Darcy's law**

$$q_w = -K_w(\nabla P_w - \rho_w g)$$
$$q_o = -K_o(\nabla P_o - \rho_o g)$$

- **Water and oil flow driven by pressure gradients**
- **Permeability tensor**

$$K_\alpha = k\, k_{r\alpha} / \mu_\alpha$$

k : intrinsic permeability tensor

$k_{r\alpha}$: relative permeability

μ_α: dynamic viscosity

$$\mathbf{k} = \mathbf{k_i} \exp[b(\phi - \phi_i)]$$

$$\frac{\partial(\phi s_\alpha \rho_\alpha)}{\partial t} + \nabla(\rho_\alpha q_\alpha + \phi s_\alpha \rho_\alpha \dot{\mathbf{u}}) = 0$$

MECHANICAL:

$$\nabla \boldsymbol{\sigma} + \mathbf{b} = 0$$
$$\boldsymbol{\sigma} = \boldsymbol{\sigma}' + S_w \cdot P_w + S_o \cdot P_o$$
$$\boldsymbol{\sigma}' = \mathbf{D} \cdot \boldsymbol{\varepsilon}$$
$$\varepsilon_v = Tr(\boldsymbol{\varepsilon}) = \nabla \mathbf{u}$$

SOLID BALANCE:

$$\frac{d\phi}{dt} = \frac{(1-\phi)}{\rho_s}\frac{d\rho_s}{dt} + (1-\phi)\frac{d\varepsilon_v}{dt}$$

HM FORMULATION

Numerical scheme to solve the coupled problem:

M: (u)

$$\nabla \boldsymbol{\sigma} + \mathbf{b} = \mathbf{0}$$

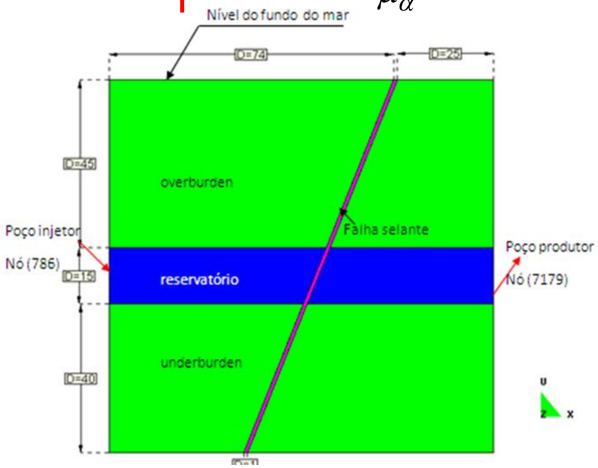
$$\boldsymbol{\sigma} = \boldsymbol{\sigma}' + p_f \cdot \mathbf{I}$$

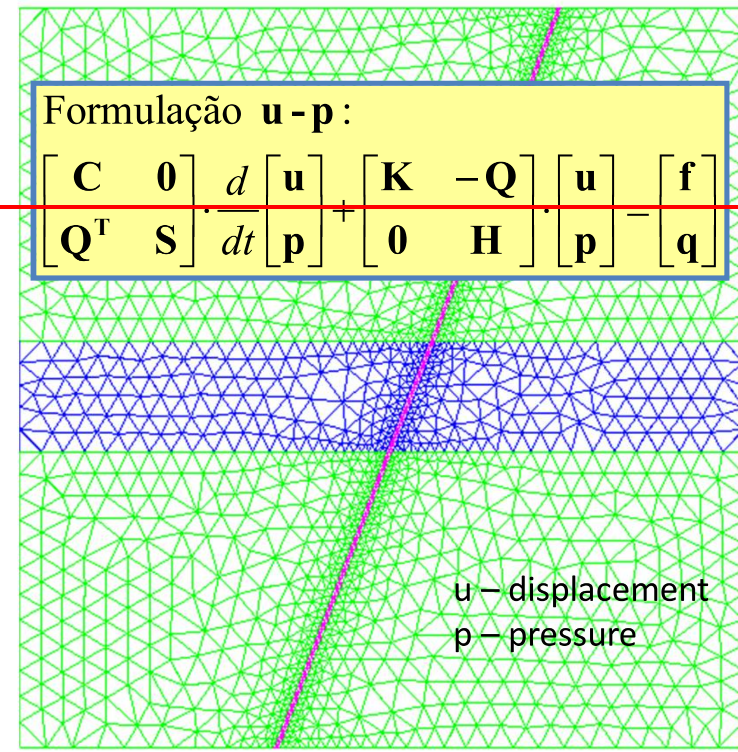
$$\boldsymbol{\sigma}' = \mathbf{D} \cdot \boldsymbol{\varepsilon}$$

H: (p)

$$\frac{\partial(\phi s_\alpha \rho_\alpha)}{\partial t} + \nabla\left(\rho_\alpha q_\alpha + \phi s_\alpha \rho_\alpha \dot{\mathbf{u}}\right) = 0$$

$$q_\alpha = -\frac{k k_{r\alpha}}{\mu_\alpha}\left(\nabla p_\alpha - \rho_\alpha \tilde{g}\right)$$





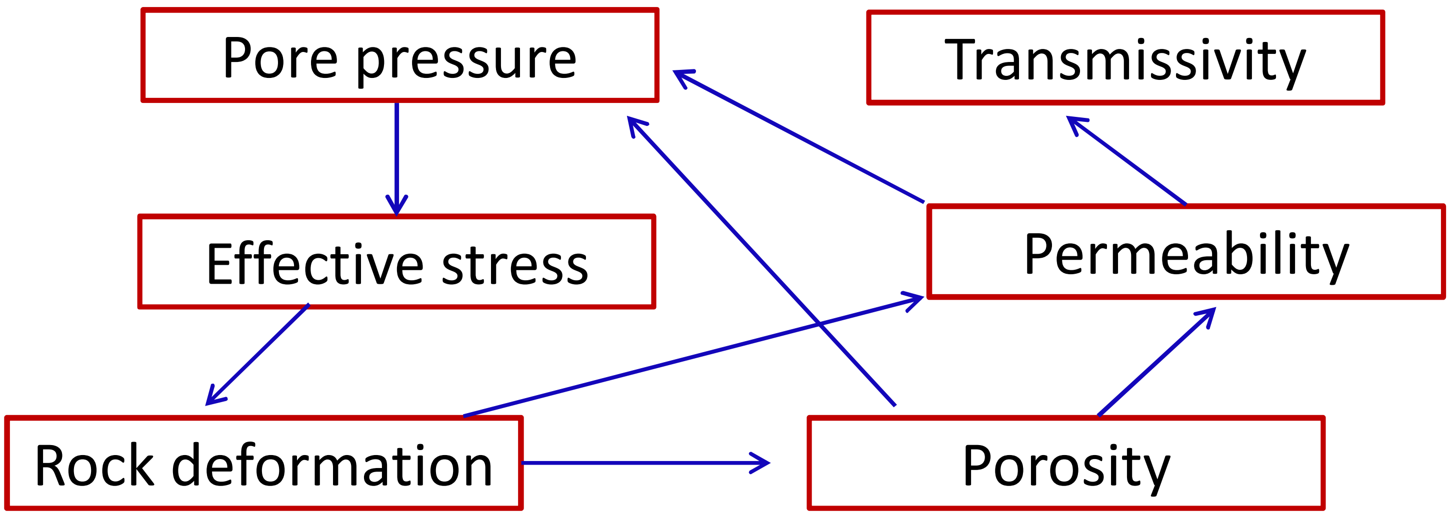
Formulação **u - p** :

$$\begin{bmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{Q}^{\mathrm{T}} & \mathbf{S} \end{bmatrix} \cdot \frac{d}{dt} \begin{bmatrix} \mathbf{u} \\ \mathbf{p} \end{bmatrix} + \begin{bmatrix} \mathbf{K} & -\mathbf{Q} \\ \mathbf{0} & \mathbf{H} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{u} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ \mathbf{q} \end{bmatrix}$$

u – displacement

p – pressure

Fluid Flow – Geomechanics Coupling

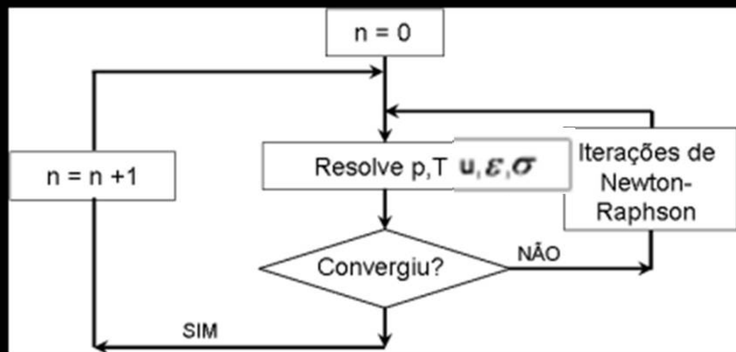


Geomechanics Simulator
FEM

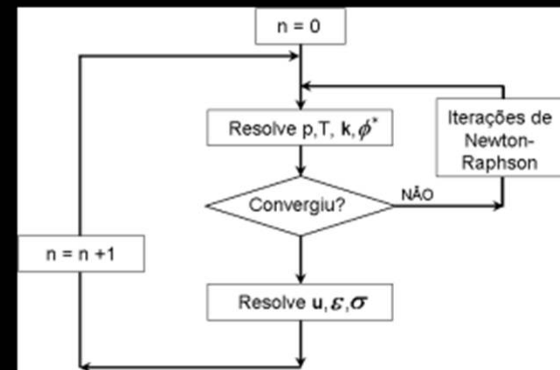
Reservoir Flow Simulator
FDM

L. C. Pereira (MSc, 2007)

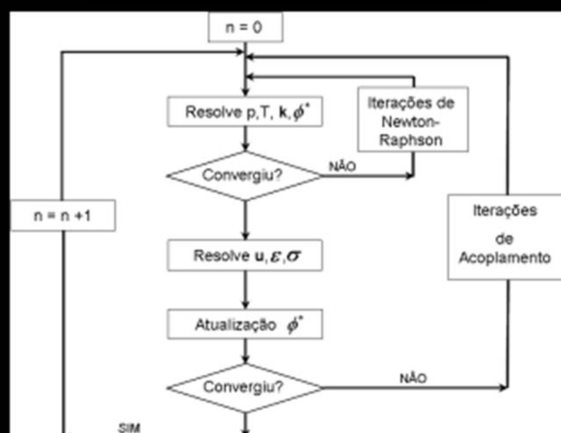
Coupling Schemes



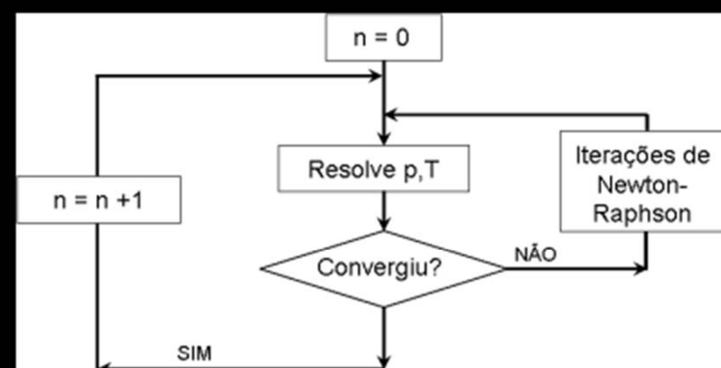
Implicit



Explicit



Iterative



Pseudo-coupling

L. C. Pereira (MSc, 2007)

Coupling Schemes

	Implicit	Iterative	Explicit	Pseudo
Computational costs				
Changes in numerical code				
Convergence control				
Accuracy				
Speed				
Iterations				

 Disadvantages

 Advantages

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Workflows to exchange parameters between individual modules

UFPE

OTC 19530

Geomechanics in Integrated Reservoir Modeling

A. (Tony) Settari, SPE, and Vikram Sen, SPE, University of Calgary

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Coupling with commercial reservoir simulators

SPE International

SPE 125760

Answers to Some Questions About the Coupling Between Fluid Flow and Rock Deformation in Oil Reservoirs

Nelson Inoue, GTEP PUC-Rio, and Sergio A. B. da Fontoura, DEC PUC-Rio

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Stress-split Method

A new locally conservative numerical method for two-phase flow in heterogeneous poroelastic media

Marcio A. Murad [a,*], Marcio Borges [a], Jesus A. Obregón [b], Maicon Correa [c]